Public Debt, iMPCs & Fiscal Policy Transmission*

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Abstract

I study how public debt shapes fiscal policy transmission. Using U.S. local projections with identified government-spending shocks, I find that output and consumption multipliers are smaller when the household-sector Treasury position is high relative to GDP. To interpret this state dependence, I develop a heterogeneous-agent New Keynesian model in which government bonds provide households with safe assets for saving and self-insurance. Higher household absorption of public debt raises the equilibrium real return, changes the distribution of liquid assets, and lowers intertemporal marginal propensities to consume (iMPCs), weakening the private-consumption amplification of government spending. Two forces drive the result. First, higher real rates alter household consumption-saving rules, with the largest effects concentrated among low-asset households. Second, the additional supply of safe assets reallocates households away from constrained, high-MPC states. A decomposition shows that the real-rate channel accounts for almost all of the change in household spending rules, while distributional reallocation remains quantitatively important. The results imply that fiscal effectiveness depends not only on the amount of public debt, but also on how public debt maps into household liquidity and equilibrium prices.

Keywords: Fiscal multipliers; Public Debt; HANK; Government Spending; iMPCs.
JEL Classification: E21; E62; H31

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1 Introduction

After the global financial crisis and again during the COVID-19 pandemic, advanced economies used large discretionary fiscal expansions to support aggregate demand and economic activity. These interventions increasingly take place in economies with elevated stocks of public debt and with large quantities of government securities held by the private sector. In the United States, for example, Treasury securities attributed to households in the Financial Accounts increased from about 3% in 2022 to 10% of annual GDP in 2026. This raises a natural question for stabilization policy: does the same fiscal intervention have different effects depending on the public-debt environment inherited when the intervention occurs? Household heterogeneity makes this question particularly relevant. Households with little liquid wealth tend to spend a larger share of additional income than households with substantial financial buffers. Public debt can affect these balance-sheet positions because government liabilities are also safe assets held by the private sector. If more public debt enters household balance sheets, households may therefore respond differently to the income generated by a subsequent fiscal expansion. The effectiveness of discretionary fiscal policy may consequently depend not only on the size of the intervention, but also on the public-debt environment in which it takes place.

This paper studies whether government-spending shocks propagate differently when households enter the shock with different exposures to public debt. I answer this question in three steps. First, I estimate state-dependent fiscal multipliers using U.S. quarterly local projections with instrumental variables (LP-IVs), with the household-sector Treasury position relative to GDP as the initial state. Second, I develop a heterogeneous-agent New-Keynesian (HANK) model in which government bonds provide households with a safe asset for saving and self-insurance, and I compare the same fiscal intervention across economies with different initial stocks of public debt. Third, I theoretically identify two distinct reasons why intertemporal marginal propensities to consume (iMPCs), which measure how consumption across horizons responds to an income innovation, change with public debt: a policy-function, or repricing, channel changes the spending response of a household at a given asset and earnings position, and a distributional channel, which changes the mass of households located at constrained and high-iMPC positions. Both channels are quantitatively relevant for fiscal multipliers.

The paper makes three contributions. First, it provides new empirical evidence on the debt state that matters for fiscal transmission. I show that fiscal transmission is weaker when public liabilities are more heavily absorbed by households. Second, it provides a quantitative mechanism for this state dependence. Higher household absorption of public debt changes household liquid balance sheets and equilibrium prices, lowers household spending responses to additional income, and weakens the private-demand amplification of fiscal policy. Third, the paper provides a new perspective on the asset-MPC trade-off in HANK models by decomposing the public debt-induced change in iMPCs into a repricing channel, which changes household spending responses at a given state, and a distributional channel, which reallocates households across asset positions. Both channels are quantitatively relevant, while within the repricing channel the equilibrium real rate accounts for most of the change in household spending rules. This decomposition also helps explain why the effect of public debt on iMPCs and fiscal transmission is nonlinear and depends on how public debt maps into household balance sheets.

Empirical facts. I first show that, in the United States, government-spending shocks have smaller effects when households enter the shock with greater exposure to public debt. I use quarterly local projections with instrumental variables and condition the response to an identified government-spending innovation on the pre-existing household-sector Treasury position relative to GDP. The specification asks whether the response to the same identified government-spending innovation varies with the initial household-sector Treasury position. The estimated cumulative output-multiplier gap between high- and low-exposure states is negative. Consumption multipliers also decline, which indicates that the state dependence is not confined to investment or other non-household components of demand. The main result is that cumulative government spending multipliers are smaller when household public-debt holdings are high. This attenuation appears in output and, importantly, in consumption. The identified variation is in the government-spending shock, not in the household Treasury position itself: the empirical exercise asks whether the same identified fiscal innovation propagates differently across predetermined household balance-sheet states.

I use the quantitative model below to isolate that comparative-static mechanism.

Finally, I use euro-area cross-country data to connect the debt-holdings measure to household balance sheets. Combining HFCS measures of hand-to-mouth behavior and MPCs with SHSS household direct holdings of domestic government securities, I document that countries where households hold more government bonds tend to have fewer constrained households and lower MPCs. This evidence disciplines the mechanism linking government-bond holdings to household spending propensities.

Quantitative mechanism. To interpret these facts, I develop and calibrate a HANK model for the U.S. economy. Households face idiosyncratic income risk and incomplete markets. Households can save in a risk-free government bond, which is the model counterpart of household exposure to safe public claims.[1] The heterogeneous agents are needed to capture a proper wealth distribution, MPCs and household spending responses. The New Keynesian block maps these heterogeneous responses into aggregate output in an economy with nominal rigidity and monetary-policy feedback. Wages are sticky, prices are flexible, monetary policy follows a Taylor rule, and fiscal policy determines the path of taxes and debt after a government spending shock. The core experiment is a comparison across steady states with different levels of government bonds, B. Across these steady states, preferences, idiosyncratic risk, productivity, steady-state government spending, wage rigidity, and policy-rule parameters are held fixed. What changes is the stock of safe public claims that households must absorb. In each steady state, the real interest rate adjusts to clear the asset market. Higher B therefore changes the economy through two objects. First, it changes the stationary wealth distribution, because households hold more safe assets in equilibrium. Second, it changes consumption-saving policy functions, because taxes and the real interest rate change. I then apply the same government spending shock in each steady state and compare the resulting paths of output, consumption, labor, taxes, debt, and interest rates.

The main fiscal experiment is deficit-financed on impact. Government spending rises unexpectedly; debt absorbs the spending increase at impact; taxes adjust only gradually afterward through a debt-feedback rule. This timing matters. The gov-

[1] In the model, B is a government-issued safe, liquid, non-contingent claim held by households and used for saving and self-insurance.

ernment spending shock raises public demand directly. Firms increase production and labor demand. Household labor income rises. The relevant question is how much of this induced income path is converted into private consumption. The answer depends on the initial stock of household-held government bonds. In a low-debt economy, many households have little liquid wealth and high MPCs. The increase in labor income generated by the fiscal expansion therefore provides stronger support to private consumption. In a high-debt economy, households are better insured and have lower spending propensities, so the same increase in labor income provides less consumption support. Consumption falls in both states because the fiscal expansion also generates a future financing burden, but the decline is larger in the high-debt economy.

To understand the mechanism, I decompose the debt dependence of household spending responses into two channels. The first is a policy-function, or repricing channel: the same household state responds differently because the equilibrium real return and tax-backed income environment are different. The second is an insurance, or distributional, channel: the same equilibrium adjustment moves households across asset states, away from constrained high-iMPC regions and toward better-insured positions. Both channels are generated by the equilibrium adjustment required to absorb a different stock of public safe claims. The relative importance of these channels is not a structural constant. The theoretical section shows that the direct real-rate component is amplified when the effective slope of household asset demand is small. This helps explain why the debt effect is strongest in low household-public-liquidity regions. However, induced reallocation remains quantitatively important, especially on impact, because the households moving away from the borrowing constraint are precisely those with high iMPCs. The incidence of the two channels differs: middle- and low-positive-asset households jointly drive repricing, while the distributional contribution is concentrated among households in and near the borrowing constraint.

A conventional MPC measures how much current consumption changes in response to additional current income. An intertemporal MPC (iMPC) instead tracks how consumption across dates responds to income received at different dates. This distinction matters for fiscal policy because a government-spending multiplier depends on the entire paths of government purchases, taxes, household income, and consumption, and not only on a single contemporaneous MPC. In the model, higher public debt substantially lowers household MPCs and iMPCs, while the decline in output multipliers is more moderate because government spending also raises demand directly. Overall, the model delivers a precise mechanism: higher public debt weakens fiscal transmission mainly by reducing private-consumption amplification, not by mechanically eliminating the direct demand effect of government spending.[2]

A key measurement issue is that headline public debt is too coarse a state variable for the mechanism studied here. The same debt-to-GDP ratio can correspond to very different domestic balance sheets depending on whether the debt is held by households, resident intermediaries, or foreign creditors. The mechanism in this paper is about public debt as a safe claim absorbed by the domestic private sector and, more importantly, by households. The empirical analysis therefore moves from the usual headline-debt state variable toward measures that are closer to the relevant balance-sheet exposure. The quantitative model then provides a disciplined environment in which this exposure can be varied directly while preferences, risk, technology, and the policy regime are held fixed.[3] In fact, in the heterogeneous agents models literature, there is a known tension between matching realistic asset holdings and matching high MPCs. If the model is calibrated to a very large liquid asset stock, households are well insured and MPCs become too low; if the model is calibrated to a smaller liquid asset stock, MPCs are higher and fiscal multipliers are more state-dependent. This paper offers a way to reinterpret this trade-off: the relevant asset target for the MPC/fiscal-transmission mechanism may not be total household wealth or headline public debt. For fiscal policy, it may be the liquid public-claim component of household-sector balance sheets, disciplined by Kaplan et al. (2018)(KMV)-style liquid wealth and by the household public-debt measure used in the empirical analysis. Overall, public-debt absorption matters because it changes the prevalence of the balance-sheet positions before the fiscal shock occurs. The relevant state is therefore the extent to which public

[2] The model isolates one mechanism. Holding preferences, demographics, productivity, idiosyncratic risk, and the policy regime fixed, a higher stock of household-absorbed safe public claims changes the asset-market-clearing real rate and household iMPCs. This does not imply that observed real rates must rise after every historical increase in public debt. Conditional on the model's maintained forces, higher public debt pushes the natural real rate upward relative to the counterfactual.

[3] This distinction is empirically relevant. Broader household-liquidity states do not reproduce the same output-consumption attenuation pattern, and the household public debt result remains when controlling for household liquid assets and total public debt relative to GDP.

safe claims enter household liquid balance sheets.

Recent changes in household public-debt absorption. Recent balance-sheet variation episodes, across countries, illustrate that the holder composition of public debt can change rapidly and can constitute a quantitatively meaningful margin. The household public-debt position is not a fixed feature of the economy. The recent U.S. balance-sheet-normalization episode provides a useful example. As the Federal Reserve reduced its Treasury holdings beginning in June 2022, the Financial Accounts household sector's Treasury position rose from approximately $0.68 trillion 2022 to $2.26 trillion one year later, and reached $3.04 trillion by 2026. Cordes and Ferris (2024) show that this sector absorbed a large portion of the Treasury securities no longer held by the Federal Reserve during the first year of balance-sheet reduction. The episode illustrates that the sectoral absorption of government debt can change rapidly and generate large movements in the household-sector public-claim state studied in this paper. Moreover, recent developments in Europe provide a complementary illustration of the household-absorption margin. As the Eurosystem reduced its government-bond footprint and sovereign issuance remained high, households became important marginal buyers of government securities. ECB sectoral holdings data show that this shift was particularly pronounced in Italy in 2023, with sizeable positive household absorption also in Spain, Portugal and Belgium, and a smaller positive contribution in Germany (Ferrara et al., 2024). These developments illustrate that household absorption of sovereign debt is neither fixed nor quantitatively negligible. The identity of the marginal buyer can change rapidly, and the resulting public claims can remain on household balance sheets after the issuance episode itself. This makes the pre-existing household public-debt position a potentially relevant state variable for subsequent fiscal transmission.

Relation to the literature and contribution. This paper first relates to the empirical literature on state-dependent fiscal multipliers. Ramey and Zubairy (2018) show that fiscal multipliers vary with the state of the economy, while Cho and Rhee (2023) document weaker fiscal transmission in high-debt economies. Most closely related, Broner et al. (2022) show that the identity of public-debt holders matters for fiscal transmission: greater foreign absorption reduces domestic crowding out and raises multipliers. Relative to this literature, I shift attention toward the household balance-sheet side of debt absorption and show that both output and consumption respond less to government-spending shocks when household public-debt exposure is high.

Second, the paper connects the literature on public debt as liquidity with heterogeneous-agent work on fiscal transmission. Woodford (1990) and Aiyagari and McGrattan (1998) emphasize that government debt provides valuable safe assets in incomplete-market economies, while Bayer et al. (2023) study the liquidity effects of public debt in an estimated HANK model. At the same time, Brinca et al. (2016, 2021) show that household wealth distributions and borrowing constraints shape fiscal multipliers.[4] I connect these insights by studying how the stock of public safe claims entering household balance sheets changes self-insurance, equilibrium prices, household spending propensities, and the transmission of a subsequent fiscal intervention.

Third, the paper relates to the intertemporal Keynesian-cross literature. Auclert et al. (2024) show that intertemporal marginal propensities to consume are key sufficient-statistic objects for fiscal transmission. I study how these spending-response objects themselves vary with the initial public-debt environment and decompose their variation into repricing and distributional channels. The contribution is therefore not only to use iMPCs to understand fiscal transmission, but to explain how a specific fiscal state variable changes the iMPC operator itself.

Outline. The remainder of the paper is organized as follows. Section 2 presents the empirical evidence on household-held public debt and state-dependent fiscal multipliers. Section 3 describes the model, and section 4 presents the calibration. Section 5 discusses the quantitative results and the role of fiscal financing. Section 6 presents the theoretical decomposition of iMPC variation into policy-function and distributional channels. Section 7 concludes.

[4] Ferriere and Navarro (2025) show that the effects of government spending depend on the distribution of the associated tax burden across households. Relatedly, Šterc et al. (2025) study how fiscal multipliers change with tax instruments, different financing, and progressivity. Compared with these papers, I focus on public debt as the main empirical state variable, and decompose how it changes iMPCs through distributional self-insurance and equilibrium prices.

2 Empirical Evidence

This section documents how fiscal multipliers vary with the absorption of public debt by households. The primary U.S. evidence uses quarterly time-series data, with household-sector public-debt relative to GDP as the state variable and a cumulative LP-IV specification with continuous state dependence. The main shock to instrument the local projections is the standard narrative military expenditure shock from Ramey and Zubairy (2018).

2.1 U.S. data and empirical strategy

To study the U.S. relationship between household-sector public-debt exposure and fiscal multipliers, I combine the U.S. time-series setup of Broner et al. (2022) with a quarterly Financial Accounts measure of Treasury securities attributed to the household sector.[5] I estimate state-dependent local projections following Jordà (2005). For output, the U.S. specification follows the quarterly cumulative continuous-interaction LP-IV design used in Ramey and Zubairy (2018). The consumption exercise below keeps the same empirical structure but changes the outcome variable, so it should be interpreted as an extension of that design. The baseline specification is

$$\sum_{j=0}^{h} y_{t+j} = \alpha_h + \beta_{1h} \sum_{j=0}^{h} g_{t+j} + \beta_{2h} \left(\sum_{j=0}^{h} g_{t+j} \times d_{t-1} \right) + \sum_{k=1}^{K} \Gamma_{k,h} X_{t-k} + \varepsilon_{t+h}, \tag{1}$$

where y is real output, g is government spending, and d_{t-1} is the lagged state variable.[6] The U.S. state variable is Treasury securities attributed to the household sector in the Financial Accounts, normalized by GDP.[7]

The empirical design closely follows the U.S. quarterly implementation in Broner

[5]Full details on sources, variable construction, and transformations are reported in the data appendix A.

[6]The state-dependent LP-IV specification conditions on the lagged household public-debt state, d_{t-1}, so the fiscal shock at t does not mechanically determine the state used in the interaction. The estimates should therefore be interpreted as initial-state conditional responses, not fixed-regime impulse responses in which the debt state is held constant throughout the horizon. The instrumented spending shock does not materially move the household public-debt-holdings/GDP over horizons 1–8.

[7]This object is a quarterly measure of household-sector public-claim absorption. It is broader than direct household Treasury holdings in the SCF: it is the long-sample public-claim state that most closely connects the empirical exercise to the household balance-sheet mechanism. Public claims are not just generic liquidity. They carry fiscal-state information that matters for fiscal transmission.

et al. (2022). All variables are scaled by potential GDP, the horizon is quarterly, and the specification is estimated in cumulative local projections. Equation (1) suppresses the first stage for compactness, but the excluded instruments are the narrative defense-news shock from Ramey and Zubairy (2018) and the Blanchard-Perotti component embedded in contemporaneous government spending, together with their interactions with the lagged debt state. This choice follows Broner et al. (2022): the defense-news shock isolates anticipated military spending innovations, while the Blanchard-Perotti component captures the contemporaneous government-spending innovation in the U.S. data.[8] The interaction term allows the cumulative multiplier to vary continuously with the lagged debt state, so that the implied multiplier at horizon h and state d is

$$m_h(d) = \beta_{1h} + \beta_{2h} d. \tag{2}$$

The figures below evaluate this object at the 90th and 10th percentiles of the U.S. household-public-debt-to-GDP distribution and report the corresponding difference.[9] A continuous specification is preferred because, in the quarterly U.S. time series, discrete high-versus-low state splits are less stable than percentile evaluation of the continuous interaction design. A potential concern is that the lagged household-debt state may partly proxy for standard business-cycle conditions. To address this concern, Appendix A.2 reports two additional checks. First, adding the verified output-gap proxy available using the U.S. dataset leaves the household-debt-state gap unchanged. Second, replacing the household-debt state with the output-gap proxy does not reproduce the benchmark output pattern.

[8]The excluded instruments identify innovations to government spending; they do not instrument the household-sector Treasury state d_{t-1}. The empirical exercise should therefore not be interpreted as estimating the causal effect of an exogenous reallocation of government bonds toward households. Instead, it asks whether the same identified fiscal innovation propagates differently across predetermined household balance-sheet states.

[9]The U.S. state variable combines a stock margin and an absorption margin. Mechanically, household-held public debt relative to GDP can be written as total public debt relative to GDP multiplied by the household share of outstanding public debt. The benchmark specification uses the ratio to GDP because it is the U.S. public- claim state that most directly captures household-sector absorption of safe government claims. Appendix A separates this stock interpretation from broader composition measures by reporting robustness exercises based on resident-held debt, domestic public debt relative to total public debt, and total public debt/GDP.

2.2 U.S. results

The main U.S. result is that fiscal multipliers are systematically smaller when household holdings of public debt relative to GDP are higher. Figure 1 reports the implied high-minus-low difference in the cumulative output multiplier from the continuous interaction specification. More precisely, the plotted object is

$$\Delta m_h \equiv m_h(d^{90}) - m_h(d^{10}) = \beta_{2h}\left(d^{90} - d^{10}\right),$$

where d^{90} and d^{10} denote the 90th and 10th percentiles of the U.S. household-public-debt-to-GDP distribution. The estimated difference is negative throughout the reported window. The gap opens early and remains economically meaningful over the main horizons, indicating that fiscal transmission is systematically weaker when the stock of public debt held by the household-sector is high relative to GDP.[10]

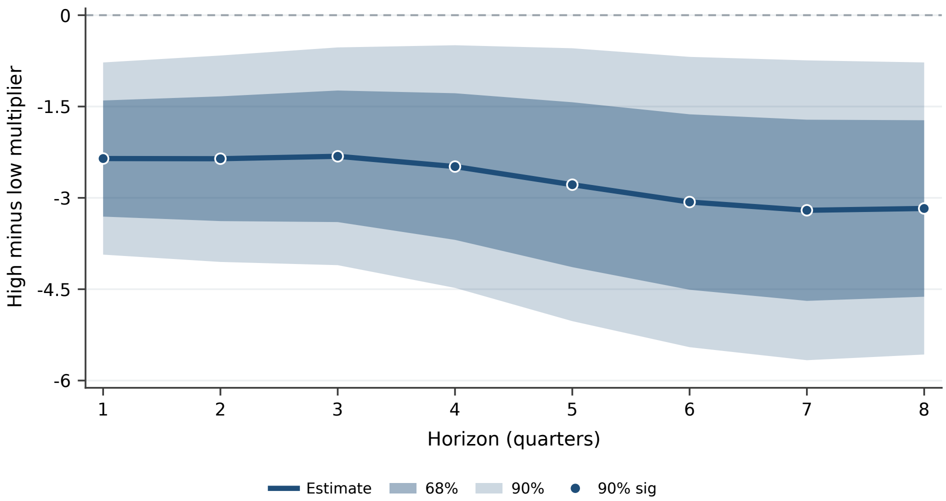

Figure 1: U.S. fiscal output multiplier: high minus low. The figure plots the difference in cumulative output multipliers evaluated at the 90th and 10th percentiles of the household-public-debt-to-GDP distribution. The state variable is household-sector public debt, normalized by GDP. A negative gap indicates that cumulative multipliers are smaller in high-household-debt states. Dark bands denote 68% confidence intervals; light bands denote 90% confidence intervals.

The consumption response provides the key mechanism evidence. Figure 2 reports the corresponding high-minus-low difference for consumption. The empirical design

[10] Appendix A.2 reports horizon-specific first-stage robustness; in the U.S. empirical exercise, the KP rk Wald F statistic remains above 15.8 at all horizons $h = 1, \ldots, 8$. The state variable is lagged, so the fiscal shock does not mechanically determine the state used in the interaction. The estimates should therefore be interpreted as fiscal transmission conditional on the initial household-public-debt state, not as fixed-regime structural impulse responses in which the debt state is held constant throughout the horizon. I verify that the identified U.S. fiscal shock does not generate a material response of the household-public-debt-to-GDP state over the main horizons.

matches the U.S. output specification, with the outcome variable replaced by real personal consumption expenditures. The gap is negative at all horizons, tracking the output result closely. This pattern is consistent with the predominant channel of the model in section 3: when households hold more safe government claims, their self-insurance capacity is higher, aggregate marginal propensities to consume are lower, and fiscal spending shocks generate weaker demand responses.

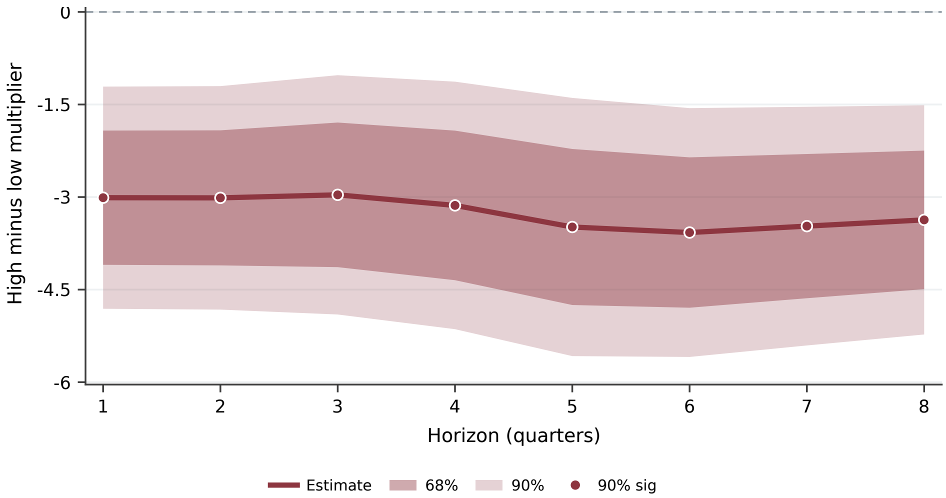

Figure 2: U.S. fiscal consumption multiplier: high minus low. The figure plots the difference in cumulative consumption multipliers evaluated at the 90th and 10th percentiles of the household-public-debt-to-GDP distribution. State variable and LP-IV design are identical to Figure 1; only the outcome is changed from output to consumption. Consumption is measured as real personal consumption expenditures normalized by real GDP. Dark bands denote 68% confidence intervals; light bands denote 90% confidence intervals.

Together, these two figures establish the main empirical fact motivating the quantitative analysis: fiscal multipliers are smaller in states with higher household public-debt holdings, and this attenuation is visible in both aggregate output and consumption. The household-holdings measure is the closest empirical counterpart to the model object B: the stock of government bonds held by households. This result is not simply a generic household-liquidity effect. Appendix A.3 reports alternative U.S. state variables as robustness checks.[11]

Household balance-sheet evidence. As a final descriptive check, I use euro-area household balance-sheet data to connect the debt-holdings measure to household liquidity positions. Combining HFCS measures of hand-to-mouth behavior with SHSS

[11] In particular, it shows that states based on broader household liquid assets, deposits, money-market funds, currency, or net worth do not reproduce the same lower-multiplier pattern. Moreover, the household public-debt state remains negative for both output and consumption after controlling for household liquidity/deposits and for total public debt relative to GDP.

household direct holdings of domestic government debt securities, I find that countries with larger household government-bond positions tend to have lower shares of hand-to-mouth households. This is a cross-country balance-sheet check showing that the household bond-holdings margin is associated with fewer liquidity-constrained households. Appendix A.5 reports the scatter plots, residualized correlations, and robustness checks.

3 Theoretical Model

This section presents the model used to study how public-debt absorption shapes the transmission of government spending shocks. The model I propose to study this question is a Heterogeneous Agent New Keynesian (HANK) model, following Auclert et al. (2024) and McKay and Reis (2016). The model features sticky wages, flexible prices, a monetary authority that follows a standard Taylor rule, and a fiscal authority that can run a balanced budget, or finance itself with deficits.

3.1 Households

Time is discrete. The economy is populated by a unit mass of households subject to idiosyncratic labor-productivity risk. Let $e_t \in \mathcal{E}$ denote the idiosyncratic earnings state and a_{t-1} beginning-of-period asset holdings. Households have preferences

$$U(c_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\varphi n_t^{1+1/\eta}}{1+1/\eta}, \tag{3}$$

where $\sigma > 0$ is the coefficient of relative risk aversion, $1/\sigma$ is the elasticity of intertemporal substitution, $\varphi > 0$ governs the disutility of labor, and $\eta > 0$ is the Frisch elasticity.

Households do not choose wages individually. Hours are determined by wage-setting unions, as in Erceg et al. (2000). The labor union setting is presented in Appendix C.

Conditional on the hours schedule N_t, the household problem is

$$V_t(e_t, a_{t-1}) = \max_{c_t, a_t} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\varphi N_t^{1+1/\eta}}{1+1/\eta} + \beta \mathbb{E}_t \left[V_{t+1}(e_{t+1}, a_t) \right] \right\} \tag{4}$$

subject to

$$c_{i,t} + a_{i,t} = (1 + r_t)a_{i,t-1} + \frac{W_t}{P_t} e_{i,t} N_t - e_{i,t} T_t, \qquad a_t \geq \underline{a}. \tag{5}$$

The fiscal adjustment is therefore distributed across households in proportion to their exogenous productivity state. Household i's effective tax burden is

$$T_{i,t} = e_{i,t} T_t. \tag{6}$$

Since productivity is normalized so that

$$\int e_{i,t} \, d\Psi_t = 1, \tag{7}$$

aggregate household tax payments equal T_t.

Equation (5) makes clear that fiscal policy affects households through three objects: the path of taxes T_t, the equilibrium real return r_t, and the aggregate supply of public safe claims B_t. Individual asset holdings a_t are household choices, and their aggregate equals the government bond supply in equilibrium. The saving asset is the public safe claim, so its aggregate supply is pinned down by government debt. Even when a spending shock is deficit-financed on impact, households respond to the induced paths of future taxes, debt, and real returns.

3.2 Production and wage setting

A representative final-goods producer operates the linear technology

$$Y_t = X_t N_t, \tag{8}$$

where X_t is aggregate productivity and N_t is effective labor input. Prices are flexible. With constant returns to scale and no price-markup wedge in the benchmark,

$$\frac{W_t}{P_t} = X_t. \tag{9}$$

Nominal rigidity is introduced through an aggregate Erceg et al. (2000)-style wage-setting block. A continuum of unions supplies differentiated labor services and adjusts nominal wages subject to wage-adjustment costs. In symmetric equilibrium, wage inflation satisfies

$$\pi_t^w = \kappa_w \left[\varphi N_t^{1/\eta} - \frac{1}{\mu_w} \frac{W_t}{P_t} C_t^{-\sigma} \right] + \beta \mathbb{E}_t \pi_{t+1}^w, \tag{10}$$

where π_t^w denotes nominal wage inflation, $\mu_w > 1$ is the steady-state gross wage markup, and $\kappa_w > 0$ controls the slope of the wage Phillips curve. Equivalently,

$$\mu_w = \frac{\epsilon_w}{\epsilon_w - 1}, \qquad \frac{1}{\mu_w} = \frac{\epsilon_w - 1}{\epsilon_w}, \tag{11}$$

where $\epsilon_w > 1$ is the elasticity of substitution across differentiated labor services.

The consumption term in Equation (10) is aggregate consumption C_t. Moreover, since the real wage satisfies $W_t / P_t = X_t$, wage and price inflation obey

$$\pi_t^w = \pi_t + \Delta \log X_t \tag{12}$$

in log-linear terms. Aggregate productivity is held fixed in the reported fiscal experiments. Hence

$$\pi_t^w = \pi_t \tag{13}$$

along the reported transitions. Sticky wages generate nominal rigidity on the labor side while preserving a simple firm block. The derivation of the wage Phillips curve is reported in Appendix C.

3.3 Monetary policy

The monetary authority follows a standard Taylor rule,

$$i_t = r_t^* + \phi_\pi \pi_t + \varepsilon_t, \tag{14}$$

where i_t is the nominal interest rate, π_t is inflation, r_t^* is the steady-state real-rate component, and $\phi_\pi > 1$ is the monetary-policy response to inflation. The real return relevant for household saving decisions is determined by the Fisher relation between the nominal rate and expected inflation. The realized real return earned in period t

on assets chosen in period $t-1$ is determined by the linearized Fisher relation

$$r_t = i_{t-1} - \pi_t. \tag{15}$$

The real return is therefore predetermined on impact with respect to a period-t fiscal innovation.

3.4 Fiscal policy, market clearing, and equilibrium

The government issues one-period risk-free public debt B_t, raises revenue through taxes, and purchases government goods G_t. I write the government budget constraint in real terms as

$$B_t = (1 + r_t)B_{t-1} + G_t - T_t. \tag{16}$$

In steady state, the government budget reduces to

$$T^{ss} = r^{ss}B^{ss} + G^{ss}. \tag{17}$$

The benchmark model studies a family of steady states indexed by the public-debt target B^{ss}. Across those steady states, government spending G^{ss} and all preference and technology parameters are held fixed. What changes endogenously are the equilibrium real rate r^{ss}, the steady-state tax level T^{ss}, and households' policy functions and stationary distribution.

Definition 1 (Stationary Equilibrium)**.** The stationary asset market clearing condition is

$$\int a\, d\Psi = B, \tag{18}$$

so the model implies $A = B$ in equilibrium.[12] The goods market clears according to

$$Y = \int c\, d\Psi + G. \tag{19}$$

A stationary competitive equilibrium is a collection of prices, allocations, house-

[12] The condition $A = B$ is an equilibrium stock condition, not a transfer rule. An increase in public bond supply does not give households assets for free: households purchase the claims through additional saving or portfolio adjustment, while the corresponding government liability is backed by future taxation.

hold policies, and a stationary distribution Ψ such that:

1. households solve (4)–(5);
2. firms satisfy optimality with flexible prices and linear technology;
3. unions set wages according to (10);
4. the monetary authority follows (14);
5. the government satisfies (16);
6. asset market clearing (18) and goods market clearing (19) hold.

3.5 What moves across steady states

The benchmark comparative statics are organized around the steady-state fiscal rule

$$T^{ss} = r^{ss} B^{ss} + G^{ss}. \tag{20}$$

Across steady states, I vary the public-debt target B^{ss} while holding government spending G^{ss}, preferences, technology, and policy-rule parameters fixed. The equilibrium real rate, the tax level, household policy functions, and the stationary distribution adjust endogenously.

The asset-market-clearing condition can be written as

$$A(r^{ss}, Z^{ss}) = B^{ss}, \tag{21}$$

where $A(\cdot)$ denotes aggregate household asset demand and Z^{ss} summarizes the after-tax income environment induced by the fiscal rule. When the fiscal-income shifter is held fixed, this expression reduces to the simpler $A(r^{ss}) = B^{ss}$.

A higher public-debt target increases the supply of safe public claims that households must hold in equilibrium. The real return adjusts to clear the asset market, while the fiscal rule changes the tax burden required to service debt. These changes shift household policy functions and the stationary distribution, thereby changing static MPCs, dynamic iMPCs, and fiscal multipliers. The quantitative sections below measure the size of each margin.

3.6 Fiscal rules on transition

The paper compares three shock-financing regimes for a one-time, unanticipated increase in government spending.

Tax-financed (balanced-budget) rule. The government raises taxes contemporaneously so that the spending shock is fully tax-financed on impact. This corresponds to an impact financing split

$$\frac{\Delta T_0}{\Delta G_0} = 1, \qquad \frac{\Delta B_0}{\Delta G_0} = 0.$$

Partial-deficit rule. The government finances the spending shock partly with taxes and partly with debt on impact. This corresponds to

$$\frac{\Delta T_0}{\Delta G_0} = 0.5, \qquad \frac{\Delta B_0}{\Delta G_0} = 0.5.$$

Deficit-on-impact rule. The government lets debt absorb the full spending shock on impact and raises taxes only later. This corresponds to

$$\frac{\Delta T_0}{\Delta G_0} = 0, \qquad \frac{\Delta B_0}{\Delta G_0} = 1.$$

For the deficit-financed benchmark experiment, taxes subsequently follow a debt-feedback rule of the form

$$T_t = T^{ss} + \phi_T(B_{t-1} - B^{ss}), \qquad \phi_\tau > 0, \tag{22}$$

combined with (16). Hence, deficit finance delays taxes; it does not eliminate them. The three regimes therefore differ in the timing of taxation, not in whether the government budget constraint is ultimately satisfied.

3.7 Computational strategy, fiscal experiment and multiplier definitions

To solve transition dynamics, I use the sequence-space Jacobian method of Auclert et al. (2021). The method linearizes the perfect-foresight equilibrium conditions around a steady state and represents each model block as a map from sequences of aggregate inputs to sequences of aggregate outputs. These block Jacobians are then composed into a single linear system imposing household optimization, wage setting, monetary policy, the fiscal rule, asset-market clearing, and goods-market clearing at every date. Given an MIT shock to government spending, the solution delivers the impulse responses of output, consumption, labor, taxes, debt, inflation, and the real return.

The benchmark transition experiment is a one-time MIT shock to government spending. Government spending rises on impact and then decays with persistence ρ_G. The main results focus on the deficit-on-impact experiment, with the tax-financed and partial-deficit rules used as comparative financing regimes.

Given the impulse responses, I define the impact multiplier as

$$m_0 = \frac{\Delta Y_0}{\Delta G_0}, \tag{23}$$

and the four-quarter cumulative multiplier as

$$m_{0:3} = \frac{\sum_{t=0}^{3} \Delta Y_t}{\sum_{t=0}^{3} \Delta G_t}. \tag{24}$$

4 Calibration

I calibrate the benchmark economy to standard U.S. moments, using the liquid-wealth logic of Kaplan et al. (2018) and the fiscal-transmission framework of Auclert et al. (2024). The discount factor and labor-disutility parameter are calibrated in the baseline economy and then held fixed when comparing steady states with different public-debt targets. Thus, changes across debt steady states reflect equilibrium price, tax, policy-function, and distributional adjustments, not re-calibration of household preferences. Flow variables are expressed relative to per-period output. Since the model is quarterly, stock variables such as public debt are reported in the calibration table in units of quarterly output; the corresponding conventional debt-to-annual-GDP ratio is $B/(4Y)$. Thus, with $Y = 1$, the benchmark value $B = 1$ corresponds to 25% of annual GDP.

4.1 Preferences and Labor

I set the standard Frisch elasticity of labor supply to 1, in line with the literature. The disutility of work and the discount factor are among the parameters calibrated to match key moments in the data. Households have CRRA preferences with coefficient of relative risk aversion $\sigma = 2$, implying an elasticity of intertemporal substitution of $1/\sigma = 0.5$, in line with Bayer et al. (2019). The discount factor β disciplines the steady-state liquid-asset target, while the labor-disutility parameter φ disciplines steady-state labor supply. For the standard calibration, $\beta = 0.967$ and $\varphi = 1.39$. These parameters are held fixed in the debt-comparative-statics exercises.

4.2 Government and Monetary Policy

I set steady-state government spending G to 16% of GDP. The persistence of the government-spending shock is set to $\rho_G = 0.9$ at a quarterly frequency, following Nakamura and Steinsson (2014). For monetary policy, I set the inflation-response coefficient to $\phi_\pi = 1.5$, following Auclert et al. (2020). The benchmark public-debt target is reported in Table 10, and the quantitative analysis studies a range of alternative debt targets around this benchmark. A central calibration issue is how to map the single liquid asset to the data. This mapping is not unique in the literature and depends on the mechanism under study. For questions about household spending responses, Kaplan et al. (2018) discipline the liquid account using household net liquid wealth, because liquid wealth is the balance-sheet object most directly related to MPCs and hand-to-mouth behavior. In a one-account model, this logic maps the model asset to the liquid self-insurance account rather than to total household wealth or headline public debt.

By contrast, fiscal-monetary applications that focus on the consolidated government budget constraint, inflation, and interest-rate arithmetic often use broader public-debt calibrations, with government debt close to annual GDP. That mapping is useful for fiscal-monetary accounting, but it assigns households a much larger stock of liquid self-insurance and therefore mechanically moves the economy toward a lower-iMPC, flatter part of the asset–MPC curve. I therefore treat the KMV-style liquid-wealth/public-claim mapping as the baseline for the MPC and self-insurance mechanism, while reporting higher public-debt calibrations as robustness exercises.[13]

4.3 Other Parameters

The nominal rigidity of the wage New Keynesian Phillips Curve is set to be 0.2. Productivity and labor demand (when the economy is in full employment) are both set to 1. Productivity is normalized to $X^{ss} = 1$. With flexible prices, the real wage satisfies $W_t/P_t = X_t$ and is therefore fixed in the reported fiscal experiments. The equilibrium real return varies endogenously with the public-debt target. Aggregate labor income varies through the endogenous hours and output response. Idiosyncratic productivity follows a Markov chain with seven states. The asset grid contains 500 points. The rest of the parameters are reported in appendix B.

5 Quantitative Results

I first compare low-debt and high-debt steady states, and then compute transition dynamics under the three fiscal rules described above.

5.1 Steady states: debt, rates, and MPCs

To understand the aggregate responses of the economy following a fiscal shock, it is essential first to examine how the economy behaves in each steady state. Each steady state represents a different equilibrium configuration of the economy, characterized by specific levels of government debt and associated macroeconomic variables. Suppose in steady-state the government follows a simple fiscal rule given by:

$$T = rB + G \tag{25}$$

In this framework, both B and G are treated as exogenous parameters. Government spending G is fixed and does not change across steady states, while the level of government debt B varies. In each steady state, the economy must satisfy the asset market clearing condition:

[13] The distinction is economically meaningful: the model results in section 5 predict strong state dependence in the empirically relevant liquid-wealth range and a flatter marginal effect when the economy is already highly liquid.

$$A(r, Z) = B \tag{26}$$

where $A(r, Z)$ represents aggregate household demand for the public safe claim, which depends on the real interest rate r and the after-tax income environment Z. This condition ensures that aggregate household demand for the public safe claim equals the stock of debt supplied by the government.

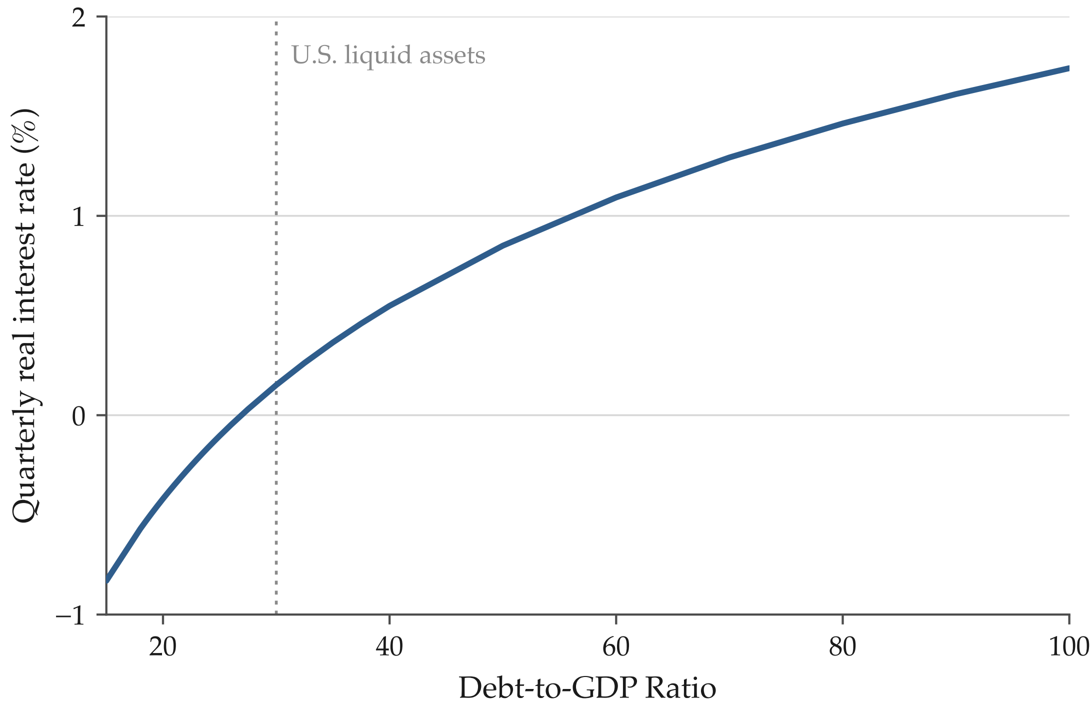

Figure 3: Determination of the interest rate in the model.

Figure 3 shows, as explained also in Achdou et al. (2022) and in the parallel work of Campos et al. (2024), how, given market incompleteness, the stock of public debt determines how much households can self-insure against negative idiosyncratic shocks and, therefore, the interest rate at which the savings market clears. As B changes, the real interest rate r adjusts endogenously to maintain market equilibrium. Specifically, an increase in the supply of government bonds requires households to absorb a larger stock of safe assets. In equilibrium, the real interest rate adjusts so that aggregate household asset demand equals the exogenous bond supply. The strength of this adjustment depends on the slope of household asset demand.[14]

Figure 4 shows how the household side changes across debt steady states. Higher

[14] See the theory in section 6.3. This price adjustment is central because it changes household saving behavior, policy functions, and ultimately dynamic iMPCs.

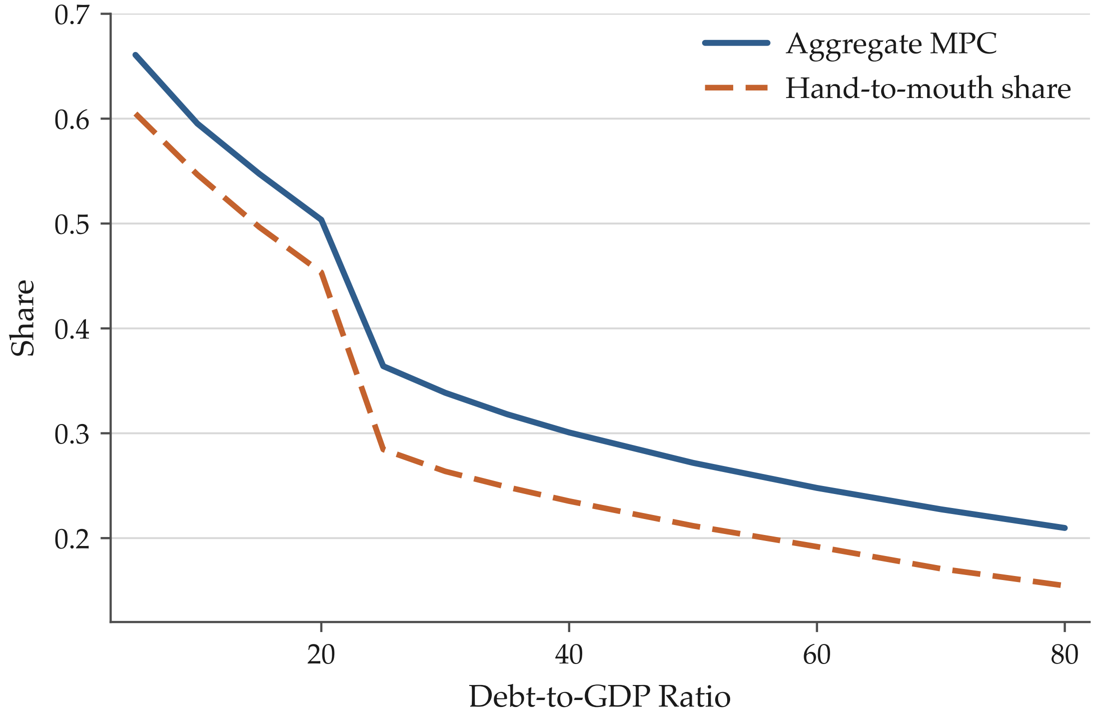

Figure 4: Changes of HtM households and aggregate MPC in the model.

public debt is associated with a lower aggregate MPC and a smaller share of hand-to-mouth households.[15] The benchmark calibration fixes household preferences and technology. I then vary the public-debt target B while holding these structural parameters fixed. The real return, taxes, household policy functions, and the stationary distribution adjust endogenously. Thus, the changes in MPCs and iMPCs below are equilibrium responses to a different public-debt environment, not the result of re-calibrating household preferences at each debt level.

Wealth reallocation and MPCs across steady states. Figure 5 documents how the wealth distribution and the consumption response vary with public debt. I normalize model liquid asset holdings by average annual labor income within each steady state and plot the household mass across bins (right axis). Overlaid is the unconditional MPC as a function of assets, constructed by averaging individual MPCs across income states at each asset level (left axis). In the high-debt steady state, the distribution shifts to the right: households hold more government bonds, while the MPC schedule declines in assets. This composition shift toward wealthier, lower-MPC households

[15] Hand-to-mouth households are computed as households at the borrowing constraint, with zero liquid assets.

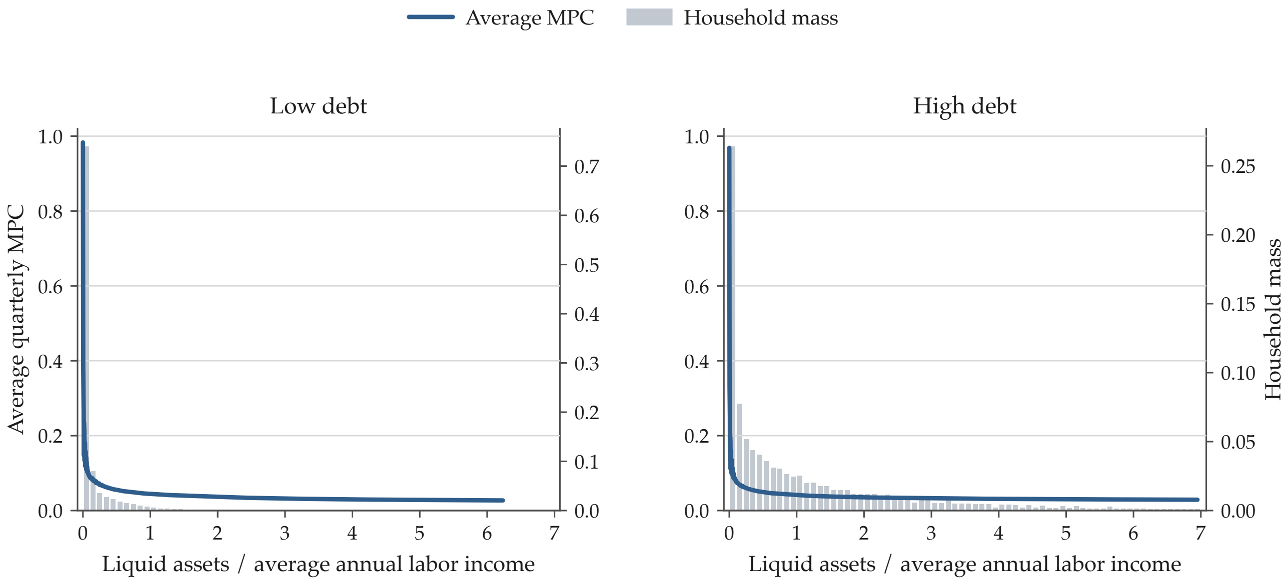

Figure 5: Wealth distribution and MPC by assets. Bars (right axis) plot the share of households in bins of liquid wealth, normalized by average annual labor income in each steady state. The line (left axis) is the unconditional MPC as a function of assets (income-averaged at each asset level). The figure illustrates the two margins behind changes in the aggregate MPC: the schedule can shift, and the distribution can place different mass along the schedule.

compresses the aggregate MPC.[16]

From composition to aggregation. Figure 6 shows the full distribution of quarterly MPCs. Relative to the low-debt economy, the high-debt distribution places more probability at low MPCs and less mass in the high-MPC tail. Together, Figures 5 and 6 establish that higher debt changes both ingredients of aggregate household spending responses: state-specific response schedules and the distribution over which they are aggregated. The figures alone do not determine the quantitative contribution of each margin. In particular, the households absorbing additional public claims need not be the households whose spending policies reprice most, and neither group need coincide with the states driving distributional reweighting. Section 6.6 therefore constructs hybrid household counterfactuals that hold either the response environment or the stationary distribution fixed. These steady-state objects form the propagation environment for the fiscal experiment: the spending shock is a flow disturbance, but its transmission depends on the pre-existing stock of public safe claims through the real rate, taxes, household balance sheets, and the dynamic iMPC operator.

[16]Figure 5 provides the visual counterpart to the two margins studied in Section 6.6. An aggregate household response can change because the state-specific response schedule changes, because the stationary distribution changes, or because both move together. The exact dynamic-iMPC decomposition below quantifies these two margins for the household spending object relevant to fiscal transmission.

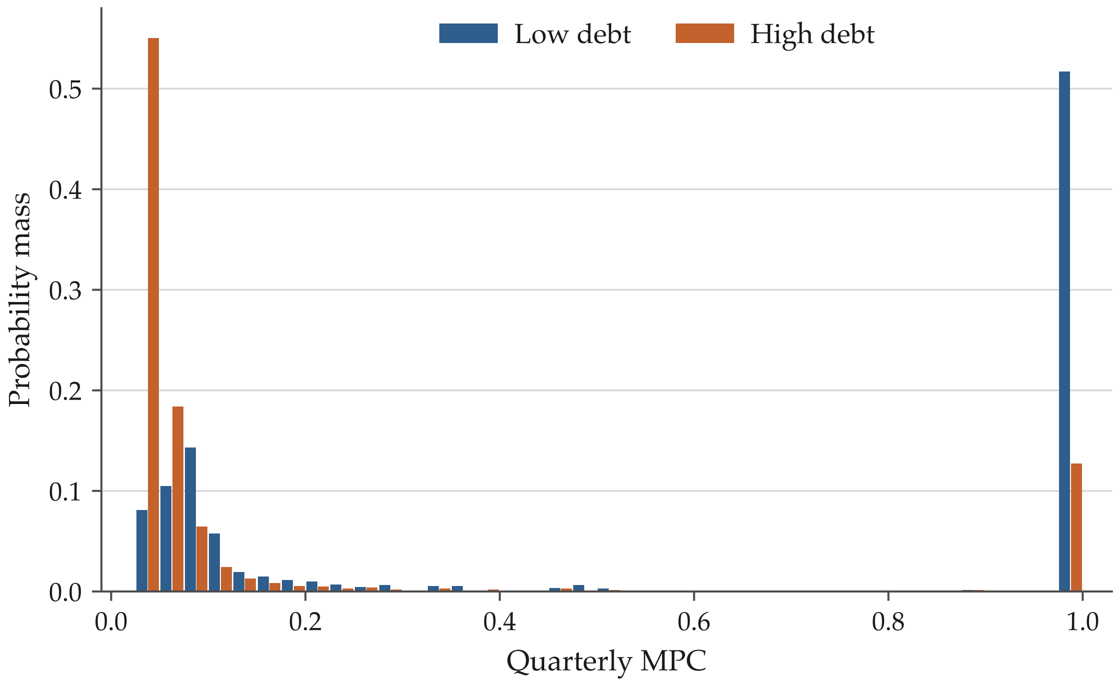

Figure 6: Distribution of quarterly MPCs: low vs. high debt. Bars show weighted probability mass by MPC bin. The high-debt steady state concentrates more mass at low MPCs and less in the upper tail, consistent with Figure 5.

5.2 Aggregate responses

I now study the transition dynamics generated by a one-time increase in government spending. The benchmark experiment in the main text uses the *deficit-on-impact* rule introduced in Section 3: the government initially finances the spending shock with debt, and taxes adjust only gradually afterward. I then compare this benchmark with the tax-financed and partial-deficit rules in Appendix D.2.

Figure 7 summarizes the main result of the model. Both the impact multiplier and the four-quarter cumulative multiplier decline with the level of public debt. Quantitatively, under the benchmark deficit-on-impact rule, the impact multiplier falls from about 0.86 in the low-debt steady state to about 0.74 in the high-debt steady state, with very similar differences for the four-quarter cumulative multiplier. Thus, the same government spending shock is less effective when the economy starts from a high-debt state.

To understand the mechanism, it is useful to begin from the benchmark impulse responses. Figure 8 reports the responses of output, consumption, labor, taxes, the real rate, and debt under the deficit-on-impact rule, comparing the low-debt and high-debt steady states. All variables are plotted as deviations from each state-specific steady

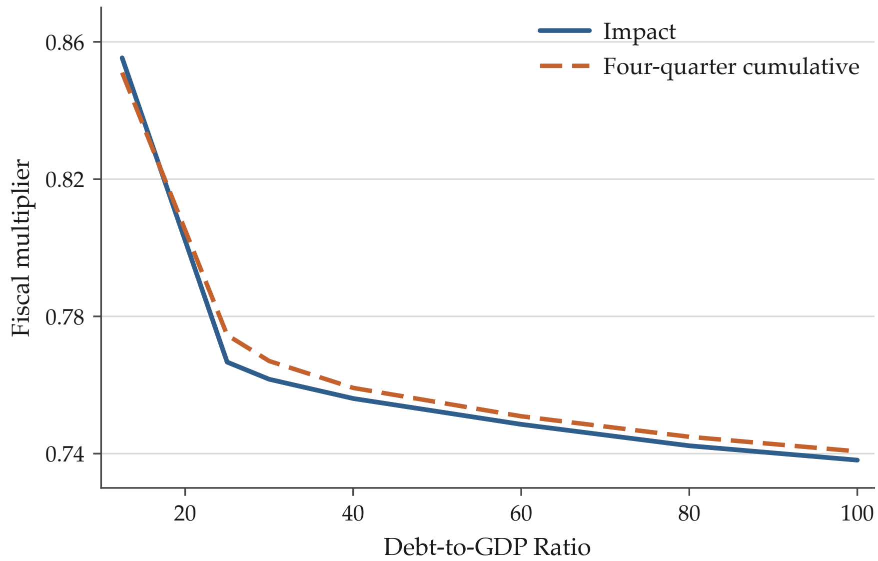

Figure 7: Impact and cumulative fiscal multipliers across debt states in the benchmark model. The figure reports the impact multiplier and the four-quarter cumulative multiplier as functions of the steady-state debt level. Higher debt is associated with smaller fiscal multipliers.

state, so the responses converge back to zero as the temporary shock dies out and the fiscal rule stabilizes debt.

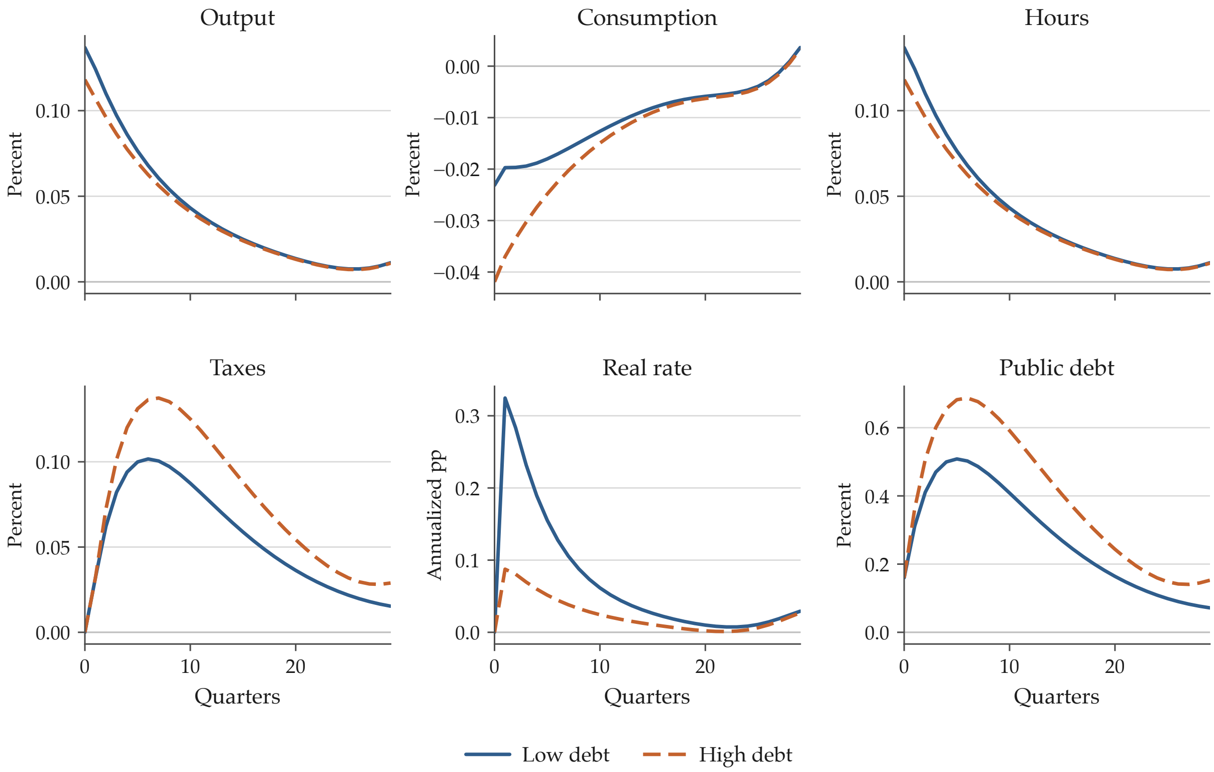

Figure 8: Impulse responses under deficit-on-impact financing: low debt versus high debt. The government spending path is identical across states by construction. Output and labor rise in both economies, but less in the high-debt state. Consumption falls on impact in both states and is more negative in the high-debt economy. Taxes do not move on impact under the benchmark rule, but rise later and by more in the high-debt state. Debt also rises more in the high-debt economy.

Chronology of the response. The transition can be understood as a sequence of linked equilibrium effects. First, the government spending shock raises public demand directly because G_t enters the goods-market identity 19. Holding private spending fixed, the increase in G_t therefore raises aggregate demand immediately. Second, firms accommodate this increase in demand through higher production. In the model,

$$Y_t = X_t N_t, \tag{27}$$

and productivity is fixed along the transition. Hence the increase in output requires an increase in labor input N_t. This is why the labor and output responses move one-for-one in Figure 8: labor is the production margin through which the shock is absorbed. Third, higher labor raises household labor income. This generates a positive private-income channel. However, this is not the only force acting on households. Under the deficit-on-impact rule, taxes do not jump at $t = 0$, but debt does:

$$\Delta T_0 = 0, \qquad \Delta B_0 = \Delta G_0. \tag{28}$$

From period $t \geq 1$ onward, taxes rise because the fiscal authority stabilizes the higher debt stock through its debt-feedback rule. A linearized version of the government budget constraint makes clear why this happens:

$$\Delta B_t \approx (1 + r^{ss})\Delta B_{t-1} + B^{ss}\Delta r_t + \Delta G_t - \Delta T_t. \tag{29}$$

Realistically, the larger is the pre-existing debt stock B^{ss}, the larger is the debt-service effect of a given interest-rate movement, and the stronger is the subsequent need for tax adjustment.

Fourth, private consumption reflects the balance between a positive income-support channel, because higher labor raises household income, and a negative fiscal and intertemporal channel, because government spending is not a transfer and the higher debt path implies future taxes and a different interest-rate path. The net consumption response is therefore not the response to a pure transfer shock. It is the response to a government spending shock that combines a demand impulse with a future financing burden.

This decomposition is the key to understanding the state dependence. In the high-debt economy, the same spending shock generates a weaker private-demand feedback. The reason is not that the shock itself differs across states, but that households enter the shock with lower MPCs and iMPCs in the high-debt steady state. As a result, households convert less of the income generated by the boom into current spending. The positive consumption support is therefore weaker, while the negative financing component remains present. This is why consumption is more negative, labor rises by less, and output responds less in the high-debt state.

What drives the state dependence? The state dependence in Figure 8 is driven by the interaction of two objects:

1. the *financing path*, which determines when taxes rise and how debt evolves after the spending shock;
2. the *household propagation structure*, summarized by the dynamic iMPC matrix around the initial steady state.

The government-spending path is the same across states. Labor and output accommodate the public-demand impulse through production. The state-dependent component comes from private consumption: lower iMPCs in the high-debt state weaken the positive income feedback and make the fiscal multiplier smaller.

5.2.1 Fiscal rules and state dependence

The appendix compares the same shock under the tax-financed and partial-deficit rules. Two main facts emerge. First, the level of the multiplier depends strongly on the financing regime: deficit finance produces the largest multipliers, tax finance the smallest, and partial deficit finance lies in between. Second, the degree of state dependence also depends on the financing rule. The low-versus-high debt gap is strongest under deficit-on-impact financing, smaller under partial deficit finance, and close to zero under tax finance. This pattern is economically intuitive. When taxes adjust immediately, the fiscal impulse is offset more quickly at the household level and there is less room for state-dependent private-demand amplification. When debt absorbs the shock first and taxes adjust only later, differences in the economy's iMPC structure matter more, and the gap between low-debt and high-debt multipliers widens.[17]

Stock versus flow of public debt. The model distinguishes between the *stock* of public debt and the *flow* of debt created by a fiscal shock. The stock of debt is the steady-state object that defines the environment in which policy operates: it determines the real return, the tax burden required to service debt, household balance sheets, and the iMPC matrix. The flow of debt is the transition response of government liabilities to the spending shock. State-dependent fiscal multipliers arise because the same flow experiment is evaluated in economies with different pre-existing debt stocks. This distinction mirrors the empirical design in Section 2, where the local projections condition on pre-shock household public-debt holdings.

5.3 Why output rises when consumption falls

The benchmark IRFs imply that output can rise even though private consumption falls. The reason is straightforward. Government spending enters aggregate demand directly, following equation 19. Hence output rises whenever the increase in public demand is larger than the fall in private demand. In the model, the spending shock first raises public absorption, which pushes output and labor upward. This raises household income. Private consumption then responds to that higher income, but the response is dampened by expected future taxes, by the debt path, and by the intertemporal incentives induced by the interest-rate path. In the low-debt state, higher MPCs make private consumption more supportive of the boom. In the high-debt state, lower MPCs weaken that supportive feedback. The result is that the high-debt economy exhibits a more negative consumption response and a smaller output multiplier, even though the government spending shock itself is identical across states.[18]

Appendix D.1 provides a more detailed timing discussion of the transition and Appendix D.2 reports the corresponding IRFs under the alternative fiscal rules.

[17]See Appendix D.2 for more details.

[18]The spending multiplier can be below one because the government-spending shock is not a pure transfer to households. In the benchmark experiment, higher government spending raises labor demand and output, but it also creates an intertemporal financing burden through future lump-sum taxes. Private consumption therefore falls on impact in the calibrated transition. Since output equals government demand plus private absorption, the increase in output is smaller than the increase in government spending, generating a multiplier below one.

5.4 Why multiplier state dependence is flatter than MPC state dependence

A striking feature of the model is that state dependence is much stronger for household spending objects than for fiscal multipliers. In the model, moving from the low-debt to the high-debt steady state lowers the static aggregate MPC from 0.57 to 0.18, a decline of 0.39. The corresponding four-quarter iMPC object falls from 0.54 to 0.23, a decline of 0.31. By contrast, the impact multiplier falls from 0.86 to 0.74, and the four-quarter cumulative multiplier from 0.85 to 0.74. Thus, the multiplier gap is only about 30% of the static MPC gap and about $35\% - 37\%$ of the four-quarter iMPC gap.

The reason this is not a contradiction is that the multiplier is not a one-to-one transformation of the static aggregate MPC. In the Intertemporal Keynesian Cross, output satisfies

$$d\mathbf{Y} = (I - \mathbf{M})^{-1}\big(d\mathbf{G} - \mathbf{M}d\mathbf{T}\big), \tag{30}$$

where $\mathbf{M}$ is the iMPC matrix. A higher debt stock lowers $\mathbf{M}$, which reduces the amplification term $(I - \mathbf{M})^{-1}$ and therefore pushes multipliers downward. But a lower $\mathbf{M}$ also reduces the tax-drag term $\mathbf{M}d\mathbf{T}$, which partly offsets the decline in the multiplier. Hence, a large decline in static MPCs or iMPCs need not translate into an equally large decline in the fiscal multiplier.

The benchmark deficit-on-impact rule makes this mechanism especially transparent. Taxes do not move contemporaneously:

$$\Delta T_0 = 0,$$

but they rise later because the fiscal rule stabilizes the higher debt stock.

Putting these pieces together yields the following interpretation of state dependence. A high-debt economy enters the shock with lower MPCs and iMPCs, so the positive private-spending support generated by the rise in labor income is weaker. At the same time, the later tax adjustment associated with debt stabilization is stronger in the high-debt state. The combination of these two margins implies that private consumption is less supportive, output rises by less, and the multiplier is smaller in the

Table 1: Benchmark tax timing under deficit-on-impact financing

	Low debt	High debt
Impact taxes T_0	0	0
Taxes over $t = 1, 2, 3$	0.17	0.20
Peak taxes	0.10	0.14
Quarter of peak taxes	6	7

Notes: Benchmark deficit-on-impact rule in the model. The table shows that deficit finance delays taxes rather than eliminating them.

high-debt state. However, because lower iMPCs also reduce the sensitivity of private spending to future taxes, the multiplier state dependence is quantitatively flatter than the state dependence in the static aggregate MPC.

This distinction is central for interpreting Figure 7 and Figure 8. The stock of public debt shapes household spending propensities, but fiscal multipliers are mediated by the full dynamic iMPC matrix, the government-spending path, and the tax path. Static MPCs are therefore useful for assessing household liquidity movements, but they are not sufficient statistics for fiscal multipliers. Section 6 formalizes this point.[19]

6 Theoretical Results

This section provides a theoretical framework for interpreting the quantitative results. The theory identifies the household-side objects through which the model operates: the dynamic iMPC operator, the effective slope of household asset demand, and the reallocation of households across asset positions.

Public debt affects fiscal transmission through two linked margins. First, a change in the debt target changes the equilibrium price–tax environment, so a household at a given state responds differently to an income innovation. I call this the policy-function, or repricing, channel. Second, the same equilibrium adjustment changes saving behavior and therefore the stationary distribution over household states. I call this the insurance (distributional) channel. The distinction is an accounting decomposition: in the model, preferences, idiosyncratic risk, technology, and the borrowing limit are fixed across debt steady states. The stationary distribution changes because households alter saving decisions in response to the new equilibrium environment.

[19] Appendix D.4 provides additional supporting tables and a heuristic scalar decomposition that illustrates this offsetting logic more explicitly.

Thus both margins are initiated by the same equilibrium adjustment required to absorb a different supply of public safe claims.

The section establishes three results. First, in a fixed-price Intertemporal Keynesian Cross framework, fiscal transmission depends on the dynamic iMPC operator. Second, changes in this operator can be decomposed locally into direct policy-function and induced distributional components. Third, asset-market clearing links the real-rate component of the direct margin to the effective slope of household asset demand. A smaller effective asset-demand slope amplifies the direct real-rate component, but whether this component dominates the distributional margin is a quantitative question.

6.1 The iMPC operator and the fixed-price IKC benchmark

Let $d\mathbf{Y}$, $d\mathbf{G}$, and $d\mathbf{T}$ denote bounded sequences of output, government purchases, and taxes. Let $\mathbf{M}(B)$ denote the household iMPC operator in the steady state indexed by public debt B, with representative entry

$$M_{t,s}(B) \equiv \frac{\partial C_t}{\partial Z_s}, \tag{31}$$

where Z_s is a one-unit perturbation to household disposable income at date s. In the fixed-price Intertemporal Keynesian Cross of Auclert et al. (2024), equilibrium output satisfies

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}(B)d\mathbf{T} + \mathbf{M}(B)d\mathbf{Y}. \tag{32}$$

Proposition 1 (IKC mapping)**.** *Suppose $\mathbf{M}(B)$ is a bounded linear operator with $\|\mathbf{M}(B)\| < 1$. For a fixed fiscal experiment $(d\mathbf{G}, d\mathbf{T})$, the unique solution to (32) is*

$$d\mathbf{Y}(B) = \big(\mathbf{I} - \mathbf{M}(B)\big)^{-1}\big(d\mathbf{G} - \mathbf{M}(B)d\mathbf{T}\big) \tag{33}$$

$$= d\mathbf{T} + \big(\mathbf{I} - \mathbf{M}(B)\big)^{-1}\big(d\mathbf{G} - d\mathbf{T}\big). \tag{34}$$

Conditional on the fiscal path, debt affects output in this benchmark through the household spending-response operator $\mathbf{M}(B)$.

Proposition 1 motivates the household object studied below. The quantitative model also contains wage-setting, monetary-policy, and endogenous-price feedbacks.

The IKC is used here to isolate the household-side sufficient statistic and to clarify the direction in which a lower iMPC operator affects fiscal transmission.

6.2 Insurance and Repricing Channels

Let $x = (a, e)$ denote the household state, where e is idiosyncratic labor productivity and a is beginning-of-period asset holdings. Let $\Theta(B)$ collect the equilibrium objects entering the household problem at debt level B. In the benchmark steady-state family, the relevant scalar inputs are the real return $r(B)$ and the after-tax income shifter $Z(B)$, so $\Theta(B) = (r(B), Z(B))$. For each horizon $h \geq 0$, let $m_h(x; \Theta)$ denote the household-level consumption response at horizon h to a one-unit disposable-income innovation on impact. Let Ψ_B denote the stationary distribution at debt level B. The corresponding aggregate response is

$$\mathcal{M}_h(B) \equiv \int m_h\big(x; \Theta(B)\big)\, d\Psi_B(x). \tag{35}$$

The collection of these responses across dates and shock dates forms the operator $\mathbf{M}(B)$.

Lemma 1 (Local decomposition)**.** *Suppose $B \mapsto \Theta(B)$ is differentiable, the family $B \mapsto \Psi_B$ is differentiable in the weak sense, and differentiation can be passed through the integral in* (35)*. Then, at every smooth debt level,*

$$\frac{d\mathcal{M}_h(B)}{dB} = \underbrace{\int \nabla_\Theta m_h\big(x; \Theta(B)\big) \cdot \Theta'(B)\, d\Psi_B(x)}_{\mathcal{P}_h(B)\ \textit{direct policy-function margin}} + \underbrace{\int m_h\big(x; \Theta(B)\big)\, d\Psi'_B(x)}_{\mathcal{D}_h(B)\ \textit{induced distributional margin}}. \tag{36}$$

The first term compares the same household states under a different equilibrium environment. The second compares different stationary distributions while holding the household response schedule fixed. In the benchmark model, the stationary distribution is itself generated by household saving under the equilibrium environment $\Theta(B)$. The distributional term should then be interpreted as the induced distributional effect of the same price-tax adjustment that moves the direct term.

Remark 1 (A price-initiated mechanism)*.* Along the equilibrium debt locus, the stationary distribution can be written as $\Psi_B = \Phi(\Theta(B))$ for fixed preferences, idiosyncratic risk, grids, and borrowing limit. Hence $\Psi'_B = D\Phi(\Theta(B))[\Theta'(B)]$. The decomposition in (36) separates the direct incidence of the new price–tax environment from the induced incidence operating through saving and the stationary distribution. It does not separate two exogenous shocks.

6.3 Asset-market clearing and the effective asset-demand slope

Let $A(r, Z)$ denote aggregate household asset demand when the household block is solved at real return r and after-tax income shifter Z. Across the benchmark steady states, government purchases and pretax aggregate income are fixed, taxes satisfy

$$T(B) = r(B)B + G,$$

and the after-tax income shifter satisfies

$$Z'(B) = -\big(r(B) + Br'(B)\big). \tag{37}$$

Asset-market clearing requires

$$A\big(r(B), Z(B)\big) = B. \tag{38}$$

Define the partial derivatives

$$A_r \equiv \frac{\partial A(r,Z)}{\partial r}\Big|_{Z}, \qquad A_Z \equiv \frac{\partial A(r,Z)}{\partial Z}\Big|_{r}, \tag{39}$$

and define the effective asset-demand slope

$$S_A(B) \equiv A_r - BA_Z. \tag{40}$$

Lemma 2 (Debt pricing). *Suppose $A(r, Z)$ is continuously differentiable and*

$$1 + r(B)A_Z > 0, \qquad S_A(B) = A_r - BA_Z > 0. \tag{41}$$

Then the equilibrium real-rate response is

$$r'(B) = \frac{1 + r(B)A_Z}{S_A(B)} = \frac{1 + r(B)A_Z}{A_r - BA_Z}, \qquad Z'(B) = -\big(r(B) + Br'(B)\big). \tag{42}$$

In particular, under the sign conditions in (41), $r'(B) > 0$. *If* $A_Z = 0$, *this reduces to*

$$r'(B) = \frac{1}{A_r}. \tag{43}$$

Lemma 2 gives the local asset-market-clearing logic. When the effective asset-demand slope $S_A(B)$ is small, a marginal increase in public safe-claim supply requires a larger adjustment in the clearing real return. When $S_A(B)$ is large, the same increase in debt can be absorbed with a smaller movement in the real return.[20]

Because $\Theta = (r, Z)$, the direct term in Lemma 1 admits the exact split

$$\mathcal{P}_h(B) = \underbrace{r'(B)\,\mathbb{E}_{\Psi_B}[\partial_r m_h]}_{\mathcal{P}_{r,h}(B)} + \underbrace{Z'(B)\,\mathbb{E}_{\Psi_B}[\partial_Z m_h]}_{\mathcal{P}_{Z,h}(B)}. \tag{44}$$

Proposition 2 (Real-rate amplification). *At a smooth debt level satisfying the assumptions of Lemma 2, the magnitude of the direct real-rate component is*

$$|\mathcal{P}_{r,h}(B)| = \left|\frac{1 + r(B)A_Z}{S_A(B)}\right| \left|\mathbb{E}_{\Psi_B}[\partial_r m_h]\right|. \tag{45}$$

Holding fixed the household rate sensitivity $\mathbb{E}_{\Psi_B}[\partial_r m_h]$ *and the fiscal-income feedback* A_Z, *a smaller effective asset-demand slope* $S_A(B)$ *amplifies the direct real-rate component of the policy-function margin.*

Proposition 2 is an amplification result. The direct real-rate component dominates the other local components only under the stronger sufficient condition

$$|\mathcal{P}_{r,h}(B)| > |\mathcal{D}_h(B)| + |\mathcal{P}_{Z,h}(B)|. \tag{46}$$

This condition is quantitative and horizon-specific. It need not hold at every debt level or for every iMPC horizon.[21]

[20] The result is a model comparative static: it holds the asset-demand environment fixed and should not be read as a claim that observed real rates must rise after every historical increase in public debt.

[21] This split applies only to the direct policy-function channel. It is not a decomposition of the full

6.4 The induced distributional margin

The induced distributional term,

$$\mathcal{D}_h(B) = \int m_h\big(x; \Theta(B)\big)\, d\Psi'_B(x),$$

depends on where the debt-induced change in the stationary distribution occurs along the household state space. If higher debt reduces the aggregate weight on constrained, high-iMPC states, the aggregate iMPC falls even when the response schedule $m_h(\cdot; \Theta(B))$ is held fixed. If instead the distributional adjustment is concentrated among households already far from the borrowing constraint, its effect on the aggregate iMPC is smaller.[22]

The distinction from the direct real-rate component is therefore conceptual. The direct component contains an explicit inverse effective asset-demand slope. The induced distributional component operates through the endogenous reweighting of household states and through the variation in household spending responses along the asset distribution.

6.5 Local measurement and finite counterfactuals

Define the hybrid aggregation functional

$$\mathcal{H}_h(\Theta, \Psi) \equiv \int m_h(x; \Theta)\, d\Psi(x). \tag{47}$$

At a smooth debt level, the local policy-function and distributional components can be approximated by hybrid counterfactuals:

$$\widehat{\mathcal{P}}_{h,\varepsilon}(B) = \frac{\mathcal{H}_h(\Theta_{B+\varepsilon}, \Psi_B) - \mathcal{H}_h(\Theta_{B-\varepsilon}, \Psi_B)}{2\varepsilon}, \tag{48}$$

$$\widehat{\mathcal{D}}_{h,\varepsilon}(B) = \frac{\mathcal{H}_h(\Theta_B, \Psi_{B+\varepsilon}) - \mathcal{H}_h(\Theta_B, \Psi_{B-\varepsilon})}{2\varepsilon}. \tag{49}$$

change in the aggregate iMPC, which also includes the induced distributional term.

[22] Appendix E.3 provides a formal Stieltjes representation of this term under additional regularity conditions. In the model, the representation can be written as an absorption-weighted change in household spending responses. Because this formula requires assumptions on boundary terms, mass points at the borrowing constraint, and monotone conditional distribution shifts, I use it as a numerical robustness.

These are partial-equilibrium household-block counterfactuals evaluated around general-equilibrium steady states. The endpoint steady states clear all markets, while the hybrid objects deliberately hold either the distribution or the household environment fixed.

Proposition 3 (Local convention-freeness)**.** *Suppose $\mathcal{H}_h$ is twice continuously differentiable in a neighborhood of the equilibrium locus. Then*

$$\widehat{\mathcal{P}}_{h,\varepsilon}(B) \to \mathcal{P}_h(B), \qquad \widehat{\mathcal{D}}_{h,\varepsilon}(B) \to \mathcal{D}_h(B) \quad \text{as } \varepsilon \to 0. \tag{50}$$

The interaction between the two local changes is second order. Hence policy-first, distribution-first, and local Shapley allocations converge to the same derivative decomposition at smooth points.

This conclusion need not hold for large endpoint comparisons or at debt levels where the household problem has kink-induced nondifferentiabilities. Accordingly, the quantitative section reports exact endpoint and Shapley objects separately when interactions are large.

6.6 From the theoretical decomposition to the quantitative mechanism

Lemma 1 decomposes the local change in an iMPC entry at a smooth debt level. The quantitative comparison, however, involves two distinct debt steady states. I therefore implement the exact finite counterpart of the local decomposition using the hybrid household functional introduced in (47).

The main household-side object is the eight-quarter cumulative iMPC,

$$\mathcal{M}^{0:7}(B) \equiv \sum_{h=0}^{7} \mathcal{M}_h(B), \tag{51}$$

which sums the household consumption response over $t = 0, \ldots, 7$. This is an object constructed from the dynamic iMPC operator. It is neither a static MPC nor, by itself, a fiscal multiplier.

For a low-debt steady state L and a high-debt steady state H, define

$$LL \equiv \mathcal{H}^{0:7}(\Theta_L, \Psi_L), \qquad HL \equiv \mathcal{H}^{0:7}(\Theta_H, \Psi_L),$$
$$LH \equiv \mathcal{H}^{0:7}(\Theta_L, \Psi_H), \qquad HH \equiv \mathcal{H}^{0:7}(\Theta_H, \Psi_H), \tag{52}$$

where

$$\mathcal{H}^{0:7}(\Theta, \Psi) \equiv \sum_{h=0}^{7} \mathcal{H}_h(\Theta, \Psi).$$

Proposition 4 (Exact finite counterpart)**.** *The total finite change in the cumulative iMPC admits the exact decomposition*

$$HH - LL = S_P + S_D, \tag{53}$$

where the order-neutral policy-function contribution is

$$S_P = \frac{1}{2}\left[(HL - LL) + (HH - LH)\right], \tag{54}$$

and the order-neutral distributional contribution is

$$S_D = \frac{1}{2}\left[(LH - LL) + (HH - HL)\right]. \tag{55}$$

Proposition 4 is the finite counterpart of Lemma 1. The Shapley allocation averages the policy-first and distribution-first orderings and assigns half of their ordering interaction to each channel. At smooth points, as the distance between the two debt states shrinks, Proposition 3 implies that the finite allocation converges to the local derivative decomposition. Appendix E.4 reports the full accounting identities and proof.

Quantitative application. I apply this decomposition to a representative comparison between steady states with public debt equal to 25% and 30% of annual GDP. This interval has numerically stable endpoints and negligible ordering dependence.

The cumulative iMPC declines by 0.0289. Policy-function repricing accounts for 48% of the decline and induced distributional reallocation for 52%. The main result is not only an exact universal 48/52 split. Rather, the two theoretically identified channels are quantitatively comparable. Appendix F.3 reports the remaining interval comparison.

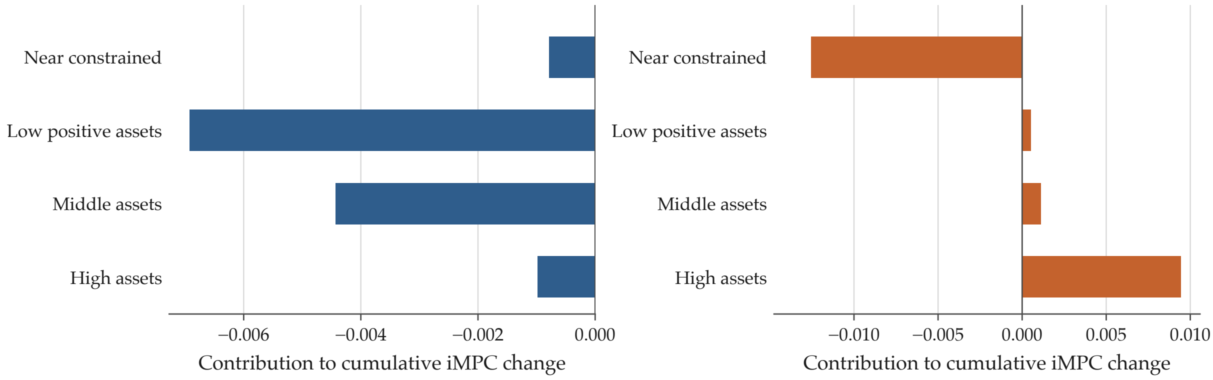

Figure 9: Household incidence of the two channels. Signed asset-status-group contributions to the decomposition. Negative values contribute to the decline in the cumulative iMPC; positive values offset it.

Figure 9 shows that the two channels operate through different regions of the household distribution. Middle-asset and low-positive-asset positions jointly drive the repricing channel. The distributional contribution is concentrated among households in and near the borrowing constraint, while some positive-asset states partly offset the decline.

The groups driving transmission are also distinct from the groups absorbing the marginal claims. High-asset positions absorb approximately 90% of the additional public safe claims, while constrained households absorb essentially none. Appendix E.5 reports the asset accounting. Within the policy-function channel, the equilibrium real-rate component accounts for 98.8% of the benchmark contribution; the full price decomposition is reported in Appendix F.1.

These results provide the quantitative counterpart to the theory. The policy-function contribution measures the first term in Lemma 1; the household incidence of that term is concentrated among middle and low-positive positions. The distributional contribution measures the second term and is driven by reallocation within the constrained region. Lemma 2 and Proposition 2 explain why the first component is real-rate led and why its importance depends on the effective slope of household asset demand.

The next result closes the argument by mapping the debt dependence of the iMPC operator into fiscal transmission.

6.7 From iMPCs to fiscal transmission

The following result makes the household-side mapping explicit within the fixed-price IKC benchmark.

Proposition 5 (Multiplier pass-through in the IKC benchmark)**.** *Let $q \equiv d\mathbf{G} - d\mathbf{T}$ be independent of B. Suppose $\mathbf{M}(B)$ is a nonnegative bounded operator with $\|\mathbf{M}(B)\| < 1$. Then*

$$\frac{d\,d\mathbf{Y}(B)}{dB} = \big(\mathbf{I} - \mathbf{M}(B)\big)^{-1}\mathbf{M}'(B)\big(\mathbf{I} - \mathbf{M}(B)\big)^{-1}q. \tag{56}$$

If $q \geq 0$, $\mathbf{M}'(B) \leq 0$ element by element, and the cumulative government-spending denominator is positive, then output responses and cumulative multipliers weakly decline with B in this benchmark.

Proposition 5 is deliberately conditional. It holds the fiscal path fixed and abstracts from the wage-setting, monetary-policy, and financing feedbacks in the full HANK model. Its role is to show the household-side sign and weighting logic: when the dynamic iMPC operator shifts downward, the private-demand amplification of government spending weakens. The full model then determines how this household-side force interacts with the financing rule, the real-rate path, and general-equilibrium feedbacks.

7 Conclusion

This paper studies how the inherited public-debt environment shapes the transmission of government spending. Public debt matters not only through fiscal space and sustainability, but also because it changes future fiscal adjustment, saving returns, and household liquid balance sheets. In the United States, output and consumption respond less strongly to identified government spending shocks when the household-sector Treasury-exposure state is high. These findings suggest that fiscal transmission depends not only on the aggregate stock of public liabilities or on conventional measures of fiscal space, but also on how households differently absorb public debt.

To interpret this evidence, I develop a Heterogeneous-Agent New Keynesian model in which government bonds provide households with a safe vehicle for saving and self-insurance. I compare the same government spending shock across steady states that differ in the initial stock of household-held public claims while holding preferences, idiosyncratic risk, technology, and policy rules fixed. A larger supply of public safe claims changes household liquid positions, the equilibrium real return, and the distribution of households across asset states. These changes lower intertemporal marginal propensities to consume and weaken the private-consumption amplification of government spending. The decline in fiscal multipliers therefore does not arise from the disappearance of the direct government-demand effect, but from a weaker endogenous response of private demand.

The theoretical analysis identifies two linked margins behind this result. Policy-function repricing changes how a household at a given state responds to income because the equilibrium intertemporal price environment changes. Induced distributional reallocation changes the weights placed on household states with different spending propensities. The quantitative decomposition shows that both margins are material. Within the policy-function component, the real rate is the central equilibrium price, while the distributional component is concentrated among households in and near constrained states. At the same time, the households absorbing most of the additional public claims are not necessarily those whose spending responses account for the decline in aggregate iMPCs. Distinguishing claim absorption from transmission incidence is therefore essential for understanding how public debt affects fiscal policy.

The results have a broader implication for the interaction between systematic and discretionary fiscal policy. The policy implication is not simply that governments should avoid debt or that high-debt economies should not use discretionary fiscal policy. Rather, headline debt alone is insufficient for assessing fiscal effectiveness. The transmission of a spending intervention depends on how the existing debt stock maps into future fiscal adjustment, equilibrium saving returns, and the distribution of household liquidity. In environments where private-demand amplification is weak, the composition and incidence of fiscal support become especially relevant. The paper does not provide a welfare ranking of debt levels or an optimal targeting rule, leaving this for future work, but it identifies the mechanisms that such an analysis would need to take into account.

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A Empirical Appendix

A.1 Additional details on the U.S. LP-IV implementation

For output, the U.S. empirical specification follows the quarterly cumulative LP-IV design of Broner et al. (2022). The main text reports the compact second-stage representation in equation (1), but the benchmark implementation uses the same excluded-instrument logic as in that paper: cumulative government spending is instrumented using the defense-news shock together with the Blanchard–Perotti component embedded in contemporaneous government spending. The state-dependent multiplier is therefore recovered from the continuous interaction specification as $m_h(d) = \beta_{1h} + \beta_{2h}d$, and the main figures evaluate this object at the 90th and 10th percentiles of the state distribution.

The baseline U.S. sample runs from the first quarter of 1950 to the last quarter of 2015, with the effective regression sample decreasing with lags and horizon. As in Broner et al. (2022), the main text focuses on second-stage multiplier estimates, while first-stage results and instrument-relevance robustness are reported here. The consumption exercise uses the same design but changes the outcome variable.

A.2 U.S. first-stage and cycle-state robustness

This appendix reports additional robustness for the U.S. LP-IV exercise. Table 2 reports horizon-specific first-stage statistics for the benchmark U.S. design. The first-stage remains strong across the reported horizon window: the KP rk Wald F statistic is above 15.8 for output and consumption at all horizons $h = 1, \ldots, 8$.

A concern with the U.S. exercise is that household public-debt holdings may proxy for the business cycle. Table 3 shows that adding this output-gap proxy leaves the household-debt-state result unchanged.

Table 4 reports the corresponding placebo exercise in which the output-gap proxy replaces the household-debt state. The placebo state does not reproduce the benchmark U.S. output pattern: the output-gap state delivers a positive high-minus-low output gap, whereas the household-debt-state benchmark delivers a negative gap. This does not eliminate all possible state-variable endogeneity concerns, but it shows

Table 2: U.S. LP-IV First-Stage Diagnostics by Horizon

h	N	KP rk Wald F	First-stage F	Min SW F	Partial R^2	Shea R^2
Panel A. Output						
1	256	318.625	285.491	483.169	0.813	0.821
2	255	61.940	122.435	149.869	0.667	0.638
3	254	28.274	84.296	58.316	0.571	0.525
4	253	24.433	64.480	38.425	0.510	0.473
5	252	26.362	50.380	36.663	0.452	0.422
6	251	22.698	41.568	33.067	0.413	0.381
7	250	19.051	35.021	29.170	0.375	0.348
8	249	15.820	30.371	25.148	0.346	0.320
Panel B. Consumption						
1	256	318.625	285.491	483.169	0.813	0.821
2	255	61.940	122.435	149.869	0.667	0.638
3	254	28.387	82.632	57.556	0.571	0.525
4	253	24.601	63.047	38.416	0.510	0.473
5	252	26.615	48.734	36.788	0.452	0.422
6	251	22.874	40.857	32.897	0.413	0.381
7	250	19.047	34.808	28.809	0.375	0.348
8	249	15.824	30.610	24.834	0.346	0.320

Table 3: U.S. Debt-State Results with Cycle Controls

Specification	Avg. gap h1–4	Avg. gap h1–8	Min KP F
Panel A. Output			
Baseline	-2.379	-2.718	15.820
+ output-gap	-2.379	-2.718	15.820
All available	-2.379	-2.718	15.820
Panel B. Consumption			
Baseline	-3.031	-3.253	15.824
+ output-gap	-3.031	-3.253	15.824
All available	-3.031	-3.253	15.824

that the benchmark U.S. result is not mechanically replicated by the available cyclical-output state.

Table 4: U.S. Cycle-Placebo Summary

Outcome	Avg. gap h1–4	Avg. gap h1–8	Neg. h	90% neg. h	90% pos. h	Min KP F
Output	0.658	0.409	0	0	1	20.173
Consumption	-0.565	-0.288	6	0	0	20.173

A.3 U.S. state-variable robustness

This appendix reports the U.S. local-projection results for three alternative state variables. All specifications use the same quarterly cumulative LP-IV design as in the main U.S. exercise. Output is the benchmark outcome; the consumption figures use the same design with consumption replacing output on the left-hand side.

The robustness exercises compare the benchmark household-sector Treasury exposure with broader measures of public-debt absorption, including resident-held public debt, the domestic share of outstanding public debt, total public debt, and liquidity-related household balance-sheet states. The purpose of these checks is not to argue that all measures are equivalent. Instead, they clarify which balance-sheet objects reproduce the output–consumption attenuation pattern emphasized in the main text. The benchmark household-sector Treasury exposure remains the best state variable because it is the closest available long-sample counterpart to the model's household safe-public-claim asset.

Domestic debt / total debt. The domestic share of outstanding sovereign debt is the closest algebraic analogue to the Broner et al. original state variable (foreign creditor share). Figures 10 and 11 report the high-minus-low output and consumption gaps. The output gap is negative throughout and large, indicating that the state-dependent pattern in output is not specific to the household-holdings measure.

Domestic debt / GDP. This measure replaces total debt in the denominator with GDP, yielding a standard debt-to-output ratio for the domestically held portion of public debt. The output gap (Figure 12) is negative at all horizons but smaller in magnitude than for the household-holdings or domestic-share measures, reflecting the lower cross-sectional spread of this series. Consumption (Figure 13) shows the same directional pattern.

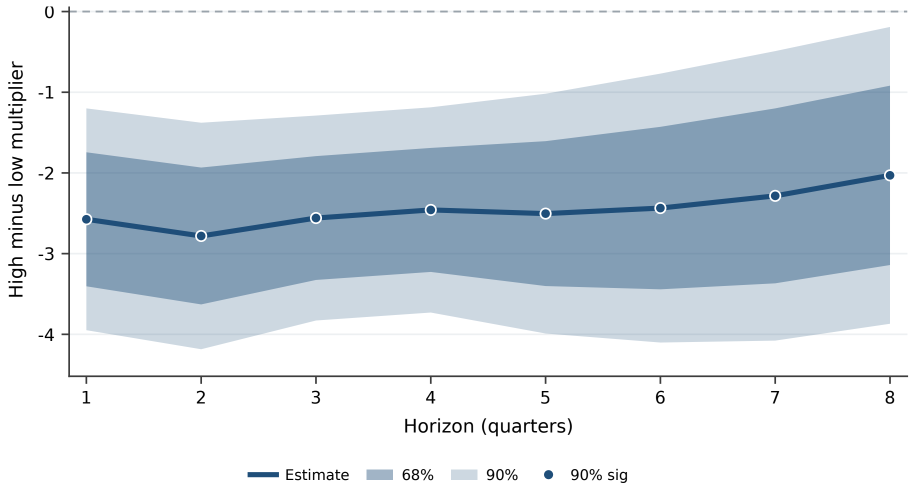


Figure 10: U.S. fiscal output multiplier, high minus low: domestic debt / total debt state variable. The state variable is the domestic share of outstanding U.S. sovereign debt (domestically held debt / total outstanding public debt), the closest algebraic analogue to the original foreign-creditor-share measure. Estimation uses the same quarterly cumulative LP-IV specification as in the main U.S. exercise. Dark bands denote 68% confidence intervals; light bands denote 90% confidence intervals.

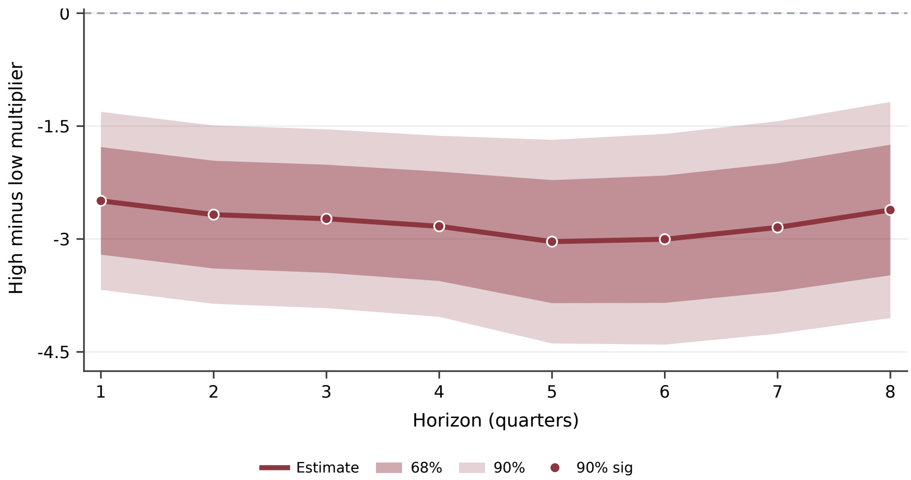


Figure 11: U.S. fiscal consumption multiplier, high minus low: domestic debt / total debt state variable. State variable and LP-IV design are identical to Figure 10. Consumption is measured as real personal consumption expenditures normalized by real GDP. Dark bands denote 68% confidence intervals; light bands denote 90% confidence intervals.

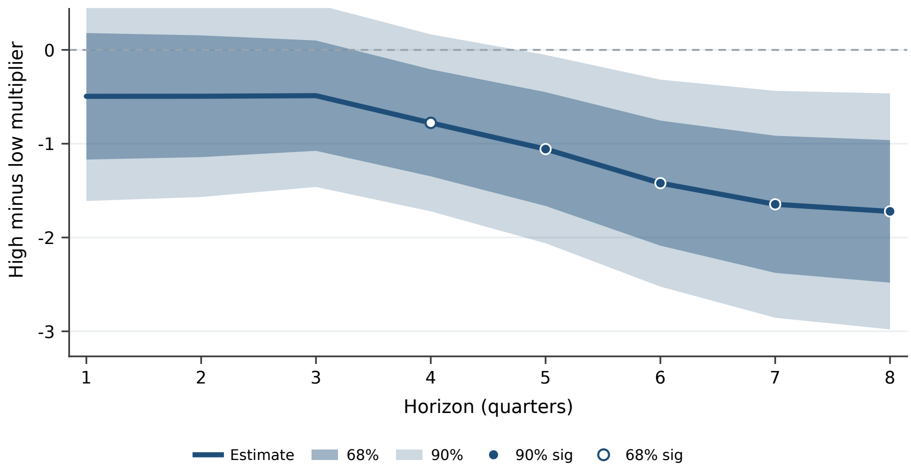


Figure 12: U.S. fiscal output multiplier, high minus low: domestic debt / GDP state variable. The state variable is total domestically held U.S. public debt normalized by GDP, a standard debt-to-output ratio for the domestically absorbed portion of sovereign debt. Estimation uses the same quarterly cumulative LP-IV specification as in the main U.S. exercise. Dark bands denote 68% confidence intervals; light bands denote 90% confidence intervals.

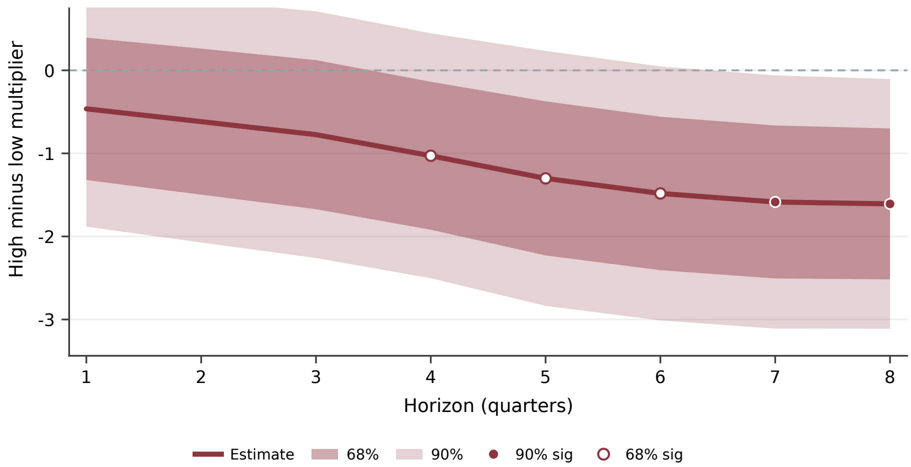


Figure 13: U.S. fiscal consumption multiplier, high minus low: domestic debt / GDP state variable. State variable and LP-IV design identical to Figure 12. Dark bands denote 68% confidence intervals; light bands denote 90% confidence intervals.

Household debt / total debt. This measure uses household direct holdings as a share of total outstanding sovereign debt. Figures 14 and 15 report the output and consumption gaps; both are negative throughout, consistent with the directional pattern in the main specification.

Other debt measures used as robustness checks. Total public debt/GDP is included as a headline-debt comparison state, not as the best mechanism-aligned state. The household and domestic-absorption states produce stronger and more stable negative gaps than headline total debt/GDP, while foreign-debt states flip sign. This supports the interpretation that the relevant margin is not public debt alone, but the domestic or household absorption of public liabilities.

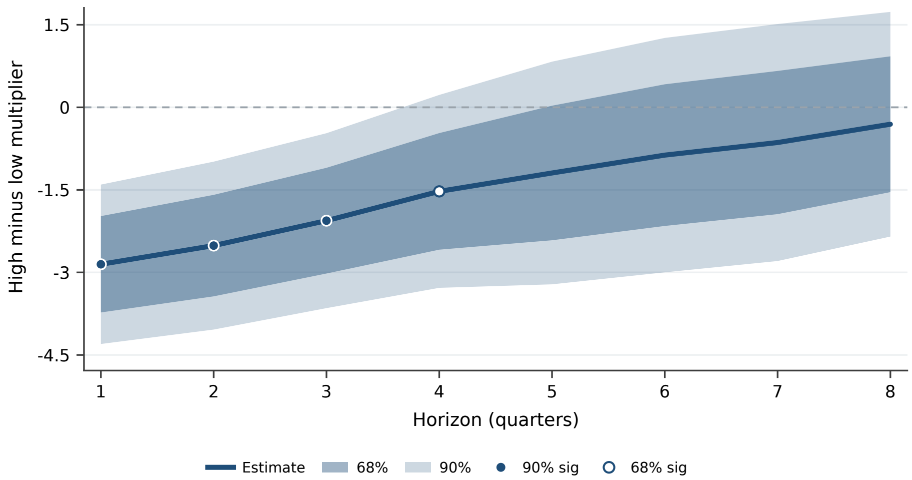


Figure 14: U.S. fiscal output multiplier, high minus low: household debt / total debt state variable. The state variable is federal debt held directly by U.S. households as a share of total outstanding sovereign debt. This is the weakest of the four state-variable measures, reflecting the narrower cross-sectional spread of the raw series. Estimation uses the same quarterly cumulative LP-IV specification as in the main U.S. exercise. Dark bands denote 68% confidence intervals; light bands denote 90% confidence intervals.

State-variable comparison. Figure 16 overlays the four high-minus-low output gaps in a single panel. Figure 17 does the same for consumption. All four measures agree on the sign of the state-dependent pattern; they differ mainly in the level of the estimated gap, which reflects differences in the empirical spread of each series at P10 and P90.

Table 5 reports the corrected common-sample averages of the U.S. high-minus-low cumulative multiplier gaps. Four conclusions emerge. First, household public debt holdings relative to GDP is the strongest state variable for mapping the U.S. evidence to the model object. Second, domestic debt relative to total debt is the

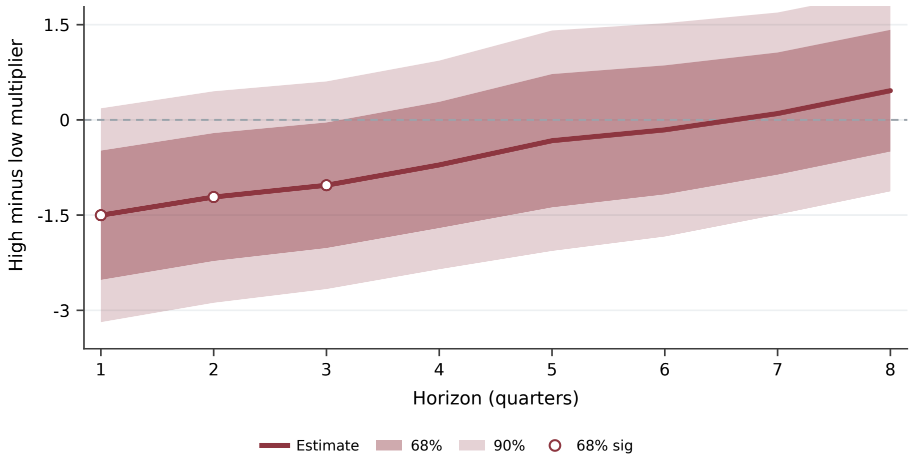


Figure 15: U.S. fiscal consumption multiplier, high minus low: household debt / total debt state variable. State variable and LP-IV design identical to Figure 14. Dark bands denote 68% confidence intervals; light bands denote 90% confidence intervals.

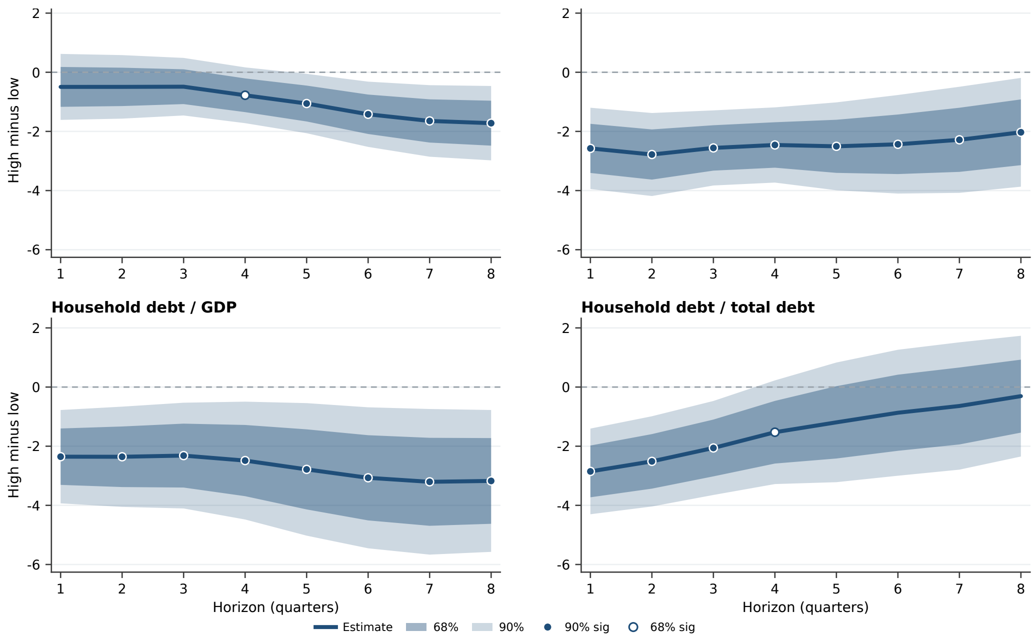


Figure 16: U.S. fiscal output multiplier gaps across all four state-variable measures. Each panel reports the high-minus-low difference in cumulative output multipliers evaluated at the 90th and 10th percentiles of the respective distribution. The four measures are: Household-sector Treasuries / GDP (main text), domestic debt / total debt (Broner-style analogue), domestic debt / GDP (aggregate ratio), and household debt / total debt (weakest measure). All estimates use the same quarterly cumulative LP-IV specification as in the main U.S. exercise. Dark bands denote 68% confidence intervals; light bands denote 90% confidence intervals.

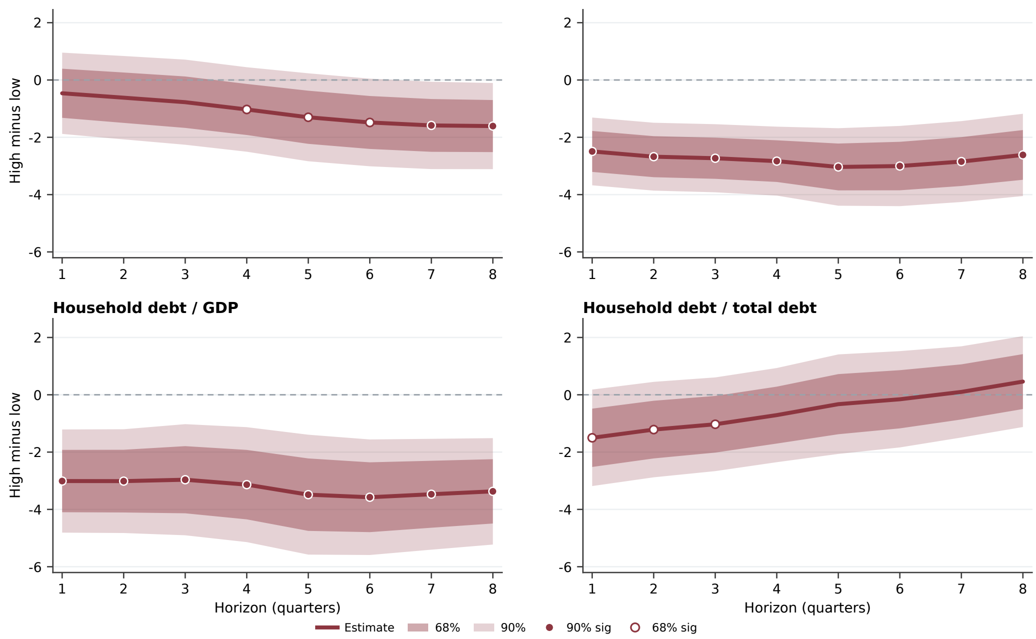


Figure 17: U.S. fiscal consumption multiplier gaps across all four state-variable measures. Same layout and design as Figure 16: high-minus-low differences in cumulative consumption multipliers evaluated at P90 and P10 for Household-sector Treasuries / GDP, domestic debt / total debt, domestic debt / GDP, and household debt / total debt. All four measures deliver a negative gap, consistent with weaker consumption responses when household or domestic debt shares are high. Dark bands denote 68% confidence intervals; light bands denote 90% confidence intervals.

strongest Broner-style robustness measure, because it is a debt-holder composition object. Third, domestic debt relative to GDP delivers the same sign pattern but smaller spreads.

Table 5: Common-sample U.S. state-variable comparison

State variable	Y avg. h=1–4	Y avg. h=1–8	C avg. h=1–4	C avg. h=1–8
Household-sector / GDP	-2.756	-3.002	-3.009	-3.225
Domestic debt / total debt	-2.778	-2.509	-2.738	-2.774
Domestic debt / GDP	-1.025	-1.521	-1.347	-1.757
Household debt / total debt	-1.871	-1.136	-1.436	-0.817

Notes: The table reports common-sample averages of the U.S. high-minus-low cumulative multiplier gap, evaluated as $P90 - P10$ from the continuous state-dependent LP-IV specification. The best model-mapping state is household-sector Treasury securities / GDP.

Table 6 summarizes the alternative-state evidence. The negative high-minus-low gap is strongest for the household public-debt-holdings proxy relative to GDP and for domestic-debt composition states. Foreign-debt states flip sign, as expected from the domestic-absorption interpretation. Headline total public debt/GDP also produces a negative gap, but it is weaker and should be treated as a scope condition.

Table 6: U.S. Alternative-State Summary

State	Outcome	Avg. gap h1–4	Avg. gap h1–8	Neg. h	90% neg. h	Min KP F	Sign
Household debt/GDP	Y	-2.379	-2.718	8	8	15.820	Negative
Household debt/GDP	C	-3.031	-3.253	8	8	15.824	Negative
Domestic debt/total debt	Y	-2.593	-2.453	8	8	28.957	Negative
Domestic debt/total debt	C	-2.683	-2.779	8	8	28.957	Negative
Domestic debt/GDP	Y	-0.564	-1.013	8	4	56.481	Negative
Domestic debt/GDP	C	-0.721	-1.108	8	2	56.481	Negative
Total debt/GDP	Y	-0.461	-1.215	8	4	29.585	Negative
Total debt/GDP	C	-1.251	-1.821	8	4	29.585	Negative
Household debt/total debt	Y	-2.238	-1.495	8	3	26.330	Negative
Household debt/total debt	C	-1.113	-0.547	6	0	24.867	Negative
Foreign debt/total debt	Y	2.593	2.453	0	0	28.958	Positive
Foreign debt/total debt	C	2.683	2.779	0	0	28.958	Positive
Foreign debt/GDP	Y	2.785	2.593	0	0	29.545	Positive
Foreign debt/GDP	C	4.679	4.606	0	0	26.637	Positive

Horizon-0 and cumulative-object robustness. The main U.S. figures plot cumulative multipliers from the continuous interaction specification. The reported high-minus-low line is the derived $P90 - P10$ difference in cumulative multipliers, not the raw interaction coefficient and not a discrete split. Table 7 reports the exact horizon-0 robustness: the implied P10, mean, P90, and high-minus-low values for output and consumption under the two main U.S. state variables.

Table 7: U.S. LP-IV diagnostics at horizon $h = 0$

State variable	Outcome	P10	Mean	P90	High$-$Low
Household debt / GDP	Output	2.829	1.322	0.268	-2.562
Household debt / GDP	Consumption	2.123	0.115	-1.289	-3.412
Domestic debt / total debt	Output	3.096	1.576	0.624	-2.471
Domestic debt / total debt	Consumption	2.216	0.822	-0.051	-2.267

Longer-horizon robustness. The cumulative objects mostly flatten rather than return to zero, while the non-cumulative analogues show more decay, especially for output. Beyond about $h = 12$, however, these robustness become materially weaker and are best treated as supporting evidence. Table 8 reports the corresponding extended-horizon cumulative and non-cumulative high-minus-low gaps together with the Kleibergen–Paap first-stage statistics.

Table 8: Extended-horizon U.S. LP-IV diagnostics ($h = 9$–16)

h	Cum. gap	Non-cum. gap	Cum. KP F	Non-cum. KP F
Panel A: Household debt / GDP — Output				
9	-3.158	-2.856	14.302	4.857
10	-3.150	-3.040	12.740	4.089
11	-3.174	-3.486	11.020	4.196
12	-3.088	-2.244	9.250	3.648
13	-3.092	-2.552	7.542	1.913
14	-2.985	-1.228	5.884	1.031
15	-2.875	1.463	4.631	0.710
16	-2.802	-0.317	4.069	0.902
Panel B: Household debt / GDP — Consumption				
9	-3.379	-3.073	14.467	5.153
10	-3.336	-2.794	12.810	4.274
11	-3.293	-3.040	11.271	3.970
12	-3.227	-2.559	9.647	3.148
13	-3.295	-3.481	7.888	1.415
14	-3.239	-2.474	6.266	0.747
15	-3.303	-3.497	4.982	0.454
16	-3.400	-2.352	4.364	0.641
Panel C: Domestic debt / total debt — Output				
9	-1.872	-0.719	25.958	13.914
10	-1.700	-0.808	23.130	8.596
11	-1.561	-0.979	21.473	9.669
12	-1.588	-0.813	19.790	7.847
13	-1.516	-0.983	14.371	4.254
14	-1.394	-0.900	10.482	2.349
15	-1.243	-0.462	7.590	1.385
16	-1.163	-0.667	6.554	1.836
Panel D: Domestic debt / total debt — Consumption				
9	-2.504	-1.766	25.497	10.598
10	-2.361	-1.521	21.720	7.782
11	-2.276	-1.735	18.177	9.014

Continued on next page

h	Cum. gap	Non-cum. gap	Cum. KP F	Non-cum. KP F
12	-2.471	-1.449	15.644	5.863
13	-2.484	-1.831	12.572	3.332
14	-2.440	-1.849	10.212	2.066
15	-2.385	-1.916	7.026	1.186
16	-2.289	-1.420	5.373	1.303

Notes: The table reports only the extended-horizon diagnostics, $h = 9$ to 16. The dedicated horizon-0 diagnostics are reported separately, and the corrected common-sample averages over $h = 1$–4 and $h = 1$–8 are reported in the main U.S. state-variable summary table. "Cum. gap" and "Non-cum. gap" denote the high-minus-low ($P90 - P10$) differences in cumulative and non-cumulative multipliers, respectively. KP F is the Kleibergen–Paap first-stage statistic. The table is intended as a diagnostic supplement rather than a headline results table.

A.4 Recent institutional evidence on public-debt absorption

Recent balance-sheet-normalization episodes provide complementary evidence that the identity of the marginal sovereign-debt buyer can change materially. This evidence is distinct from the state-dependent LP-IV results: it documents the flow of public securities onto private balance sheets.

United States. The Federal Reserve began reducing its balance sheet in June 2022. Between June 2022 and May 2024, its Treasury holdings declined by about \$1.3 trillion. Using Financial Accounts holdings, SOMA data, Treasury issuance, and Enhanced Financial Accounts hedge-fund positions, Cordes and Ferris (2024) study which sectors absorbed the securities no longer held by the Federal Reserve. Over the first year of the runoff, the household sector purchased a substantial portion of these securities. Figure 18 shows husehold-sector Treasury holdings rose from approximately \$0.85 trillion in the second quarter of 2022 to \$2.31 trillion in the second quarter of 2023. Measured domestic hedge-fund holdings remained close to \$0.2 trillion, implying that domestic hedge funds do not account for the sharp household-sector increase.

Euro area. The euro-area evidence provides a cleaner measure of direct household exposure. Using ECB Securities Holdings Statistics, Figure 19 reports the market value of government debt securities held directly by households and NPISH as a share of

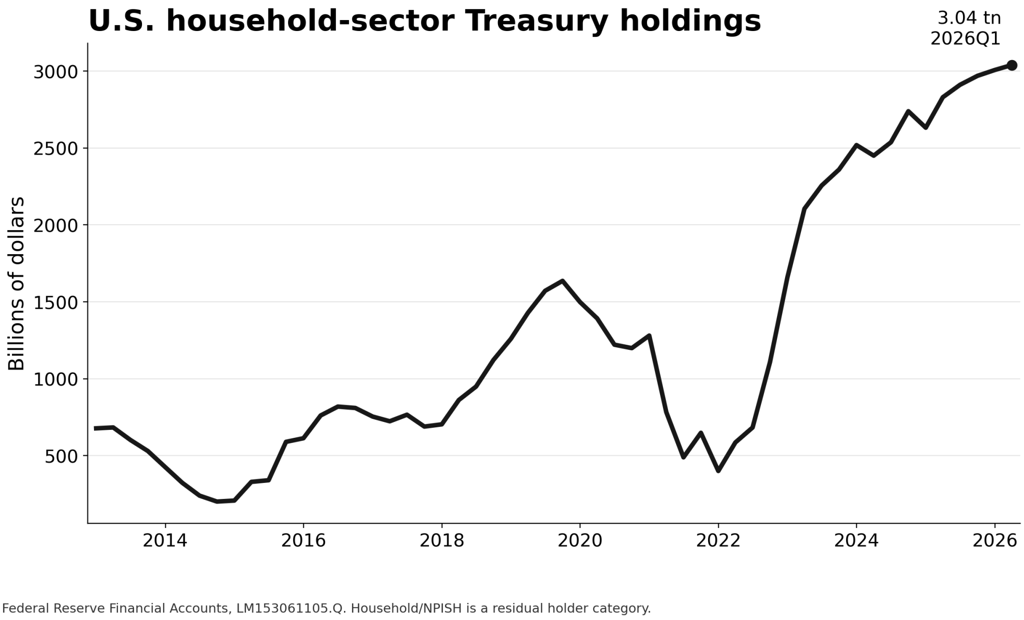
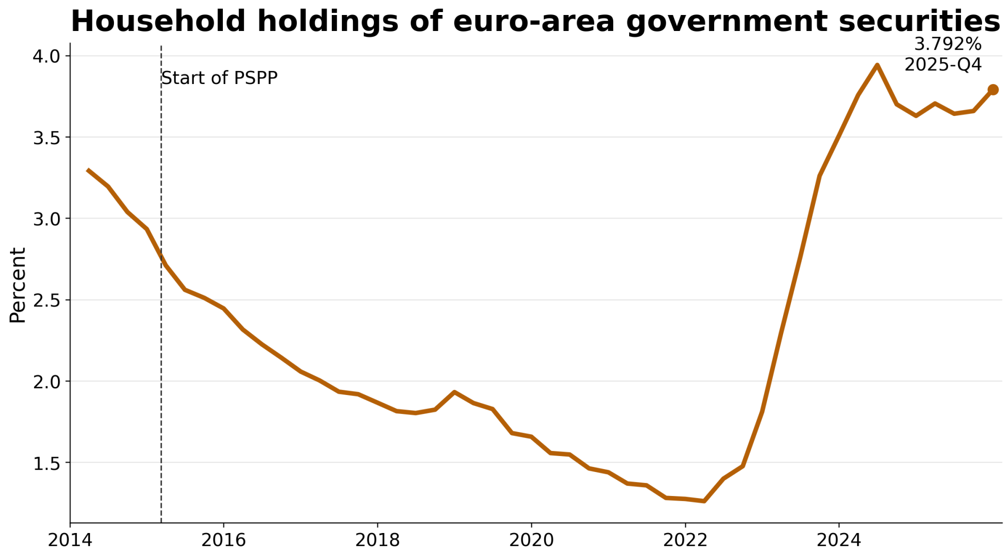


Figure 18: U.S. household-sector Treasury holdings. The figure reports the market value of Treasury securities attributed to households and nonprofit institutions in the Federal Reserve Financial Accounts. The household and NPISH sector is a residual holder category and should not be interpreted as literal direct retail ownership. The figure documents the magnitude and time variation of the inherited household-sector public-claim state. Source: Federal Reserve Financial Accounts.

outstanding euro-area government securities. Household exposure declined during the Eurosystem asset-purchase era, reached a low around 2022, and then increased sharply as sovereign yields rose, dedicated retail products became more attractive, and the Eurosystem reduced its market footprint (Ferrara et al., 2024).

Direct household holdings returned to approximately 3.5 percent of outstanding euro-area government securities by the end of 2023 and remained elevated thereafter, reaching about 3.8 percent in the fourth quarter of 2025. The persistence of the stock is important for the empirical interpretation of the paper. Household purchases can be concentrated in particular periods, while the resulting stock of public claims can remain on household balance sheets after marginal purchases normalize.

These two episodes establish a relevant fact: household and domestic absorption of sovereign debt is neither mechanically fixed nor quantitatively negligible. Central-bank balance-sheet policies, government issuance strategies, relative returns, and investor demand can produce large changes in the sectoral distribution of government securities.



Sources: ECB SHS/SHSS and QSA. Direct household/NPISH holdings; market value.

Figure 19: Household holdings of euro-area government securities. The figure reports direct household and NPISH holdings of euro-area government debt securities as a share of outstanding government securities, at market value. Household exposure declined during the Eurosystem asset-purchase period, increased sharply after 2022, and remained elevated through 2025:Q4. Sources: ECB SHS/SHSS and QSA.

A.5 Household government-bond holdings and household balance sheets

This appendix provides additional descriptive evidence linking household holdings of government securities to household balance-sheet positions across euro-area countries. It complements the household-balance-sheet paragraph in the main empirical section and the mechanism developed in Sections 3–5. The main evidence on state-dependent fiscal transmission comes from the U.S. LP-IV estimates in Section 2.2. The cross-country evidence in this appendix instead asks whether the household bond-holdings object that motivates the model has empirical content for household liquidity positions.

I combine two data sources. Household direct holdings of domestic government debt securities are measured from the ECB Securities Holdings Statistics by Sector (SHSS), focusing on the household and NPISH sector and scaling holdings by annual nominal GDP. Household liquidity positions are measured using the Household Finance and Consumption Survey (HFCS). In particular, I compute country-level hand-to-mouth (HTM) shares using a liquid/illiquid wealth classification and HFCS population weights. The resulting country-level dataset links household direct holdings of domestic government securities to the prevalence of liquidity-constrained households.

Figure 20 reports the main descriptive relationship. The left panel plots the raw cross-country association between household direct holdings of domestic government debt securities and the HTM share. The right panel reports the corresponding Frisch–Waugh–Lovell residualized relationship after partialling out age structure, mortgage participation, and loan-to-value ratios. In both panels, the relationship is negative: countries where households hold more domestic government securities tend to have fewer hand-to-mouth households.

I do not use this evidence to identify the causal effect of government-bond holdings on MPCs or fiscal multipliers. Instead, I use it as a balance-sheet consistency check: the countries in which households directly hold more government securities are also countries in which fewer households appear liquidity constrained. The evidence is nevertheless useful for the mechanism. In the model, a higher stock of safe government claims held by households improves self-insurance and reduces the mass of high-MPC, liquidity-constrained households. The HFCS/SHSS patterns are consistent with this mechanism: the directly observed household bond-holdings margin is negatively associated with hand-to-mouth behavior. This supports the interpretation of household bond holdings as a relevant balance-sheet state variable, while leaving the causal interpretation of fiscal transmission to the LP-IV exercises and the quantitative model.

As a further check, Table 9 stacks the 2013 and 2023 HFCS waves for the common two-wave country sample and estimates regressions of the HTM share on resident-held public debt. The country fixed-effects specification removes time-invariant country heterogeneity and asks whether, within a given country, higher resident-held public debt is associated with lower household constraints. The coefficient remains negative in the total-HTM specification.

Overall, the HFCS/SHSS evidence supports a narrow conclusion. Countries with larger directly observed household positions in domestic government securities tend to have fewer liquidity-constrained households, and broader domestic absorption is positively related to the household bond-holdings margin.

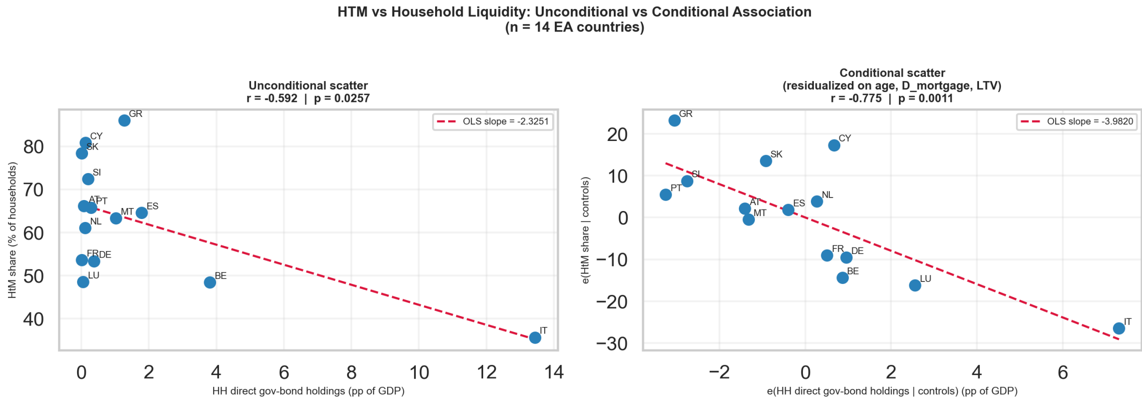


Figure 20: Household government-bond holdings and hand-to-mouth shares. The figure reports descriptive euro-area cross-country relationships between household direct holdings of domestic government debt securities and the share of hand-to-mouth households. Household government-bond holdings are measured from SHSS for the household and NPISH sector and scaled by annual GDP. Hand-to-mouth shares are computed from HFCS microdata using household population weights and a liquid/illiquid wealth classification. The left panel plots raw country-level values. The right panel reports Frisch–Waugh–Lovell residuals after partialling out age structure, mortgage participation, and loan-to-value ratios. Negative values in the right panel are residual deviations from predicted holdings or hand-to-mouth shares; they do not represent negative government-bond holdings. Dashed lines are OLS fits. Panel titles report the corresponding Pearson correlations and p-values.

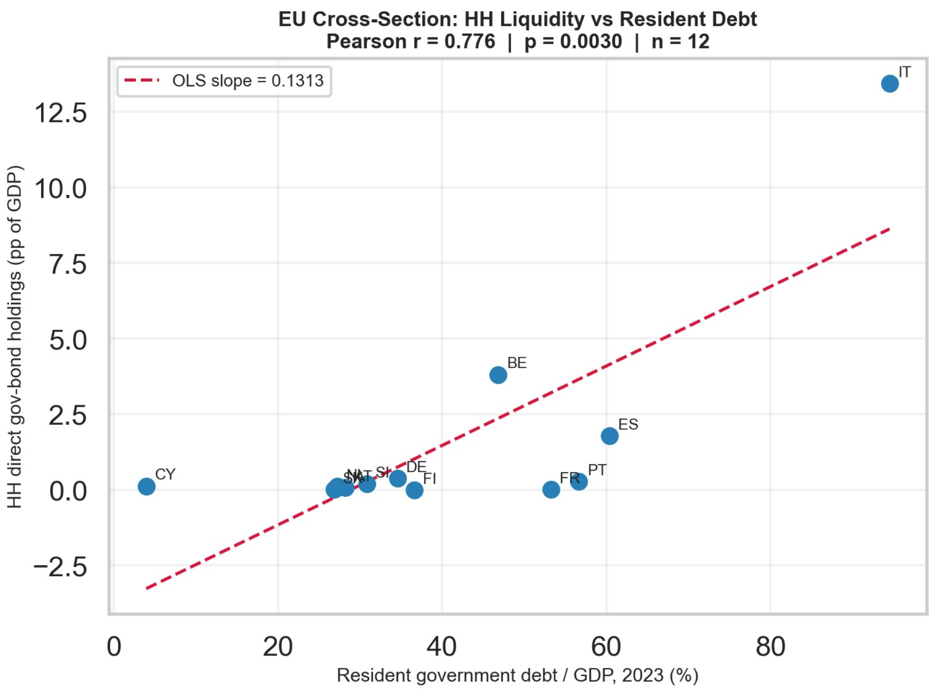


Figure 21: Resident-held debt and household government-bond holdings. Each point is one euro-area country with both resident-debt and SHSS household-holdings data in the merged sample. The horizontal axis is resident-held sovereign debt relative to GDP, assembled from official sovereign-debt statistics in the merged cross-country dataset. The vertical axis is household direct holdings of domestic government debt securities relative to annual GDP, measured from SHSS for the household and NPISH sector. The dashed line is the OLS fit. This figure is a mapping result: it shows that broader domestic absorption and the household bond-holdings margin move together across countries.

Table 9: 2013/2023 panel: resident-held debt and the HTM share. The table reports pooled and fixed-effects estimates from the stacked two-wave panel. The main within-country specification uses the common 10-country sample, giving 20 country-year observations. The dependent variable is the HTM share; the regressor of interest is resident-held public debt relative to GDP. Heteroskedasticity-robust (HC1) standard errors are reported in parentheses.

Specification	$\hat{\beta}$	SE	N	R^2
(1) Pooled OLS	-0.0014	(0.0010)	20	0.089
(2) Year FE	-0.0014	(0.0010)	20	0.105
(3) Country FE (within)	-0.0023^{**}	(0.0011)	20	0.254
(4) Pooled OLS - W-HTM	-0.0024	(0.0017)	20	0.081
(5) Country FE - W-HTM	-0.0009	(0.0070)	20	0.001

Notes: HTM specifications use the common 10-country two-wave sample. Country FE uses within-country demeaned variables. W-HTM denotes wealthy hand-to-mouth households. HC1 robust standard errors are reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

B Full model parameters Calibration Description

Parameter	Description	Value
Household		
β_1	Discount factor 1	0.967
ϕ	Disutility of labor	1.39
	Inverse IES	
0.5		
$\underline{b}$	Borrowing constraint	0.0
η	Frisch Elasticity	1
ρ_ε	Autocorrelation of earnings	0.966
σ_ε	Cross-sectional std of log earnings	0.50
Government		
θ_0	Income tax level	0.788
θ_1	Income tax progressivity	0.137
G	Government spending	0.16
B^g	Bond supply	1
ϕ_π	Taylor rule coefficient on inflation	1.5
Grid Parameters		
n_ε	Points in Markov chain for ε	7
n_A	Points on liquid asset grid	500
Other Parameters		
κ_w	Slope Wage NKPC	0.2

Table 10: List of calibrated parameter values.

C New-Keynesian Wage Phillips Curve Derivation

This appendix derives the wage New-Keynesian Phillips curve used in the quantitative model. The wage-setting block follows the differentiated-labor framework of Erceg et al. (2000), adapted to the aggregate heterogeneous-agent environment as in Auclert et al. (2024). Individual households solve their consumption-saving problem taking aggregate hours and wages as given, while a continuum of unions sets differentiated nominal wages.

There is a continuum of labor varieties indexed by $k \in [0,1]$. Aggregate effective labor is given by the CES aggregator

$$N_t = \left[\int_0^1 N_{k,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} \, dk \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \qquad \epsilon_w > 1, \tag{57}$$

where ϵ_w is the elasticity of substitution across labor varieties. Cost minimization by the labor aggregator implies demand for the labor service supplied by union k:

$$N_{k,t} = \left(\frac{W_{k,t}}{W_t} \right)^{-\epsilon_w} N_t, \tag{58}$$

where $W_{k,t}$ is the nominal wage set by union k, and

$$W_t = \left[\int_0^1 W_{k,t}^{1-\epsilon_w} \, dk \right]^{\frac{1}{1-\epsilon_w}} \tag{59}$$

is the aggregate nominal wage index. It follows that

$$\frac{\partial N_{k,t}}{\partial W_{k,t}} = -\epsilon_w \frac{N_{k,t}}{W_{k,t}}. \tag{60}$$

Union k chooses its nominal wage taking aggregate quantities and prices as given. The relevant wage-setting problem can be written as

$$\max_{\{W_{k,t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \Lambda_{t+s} \frac{W_{k,t+s}}{P_{t+s}} N_{k,t+s} - \varphi \frac{N_{k,t+s}^{1+1/\eta}}{1+1/\eta} - \frac{\psi_w}{2} \left(\Pi_{k,t+s}^{w} - 1 \right)^2 \right\}, \tag{61}$$

subject to Equation (58), where

$$\Pi_{k,t}^{w} \equiv \frac{W_{k,t}}{W_{k,t-1}} \tag{62}$$

denotes gross wage inflation. The first term in Equation (61) is real wage income evaluated in marginal-utility units, the second term is the disutility of labor, and the third term is a quadratic nominal wage-adjustment cost.

The aggregate wage-setting block uses

$$\Lambda_t = C_t^{-\sigma}, \tag{63}$$

where C_t is aggregate consumption. This is the aggregate marginal-utility object entering the numerical implementation.

Using Equation (60), the derivative of real wage income with respect to the union's wage is

$$\begin{aligned} \frac{\partial}{\partial W_{k,t}} \left(\frac{W_{k,t}}{P_t} N_{k,t} \right) &= \frac{1}{P_t} \left[N_{k,t} + W_{k,t} \frac{\partial N_{k,t}}{\partial W_{k,t}} \right] \\ &= (1-\epsilon_w) \frac{N_{k,t}}{P_t}. \end{aligned} \tag{64}$$

The corresponding marginal labor-disutility term is

$$-\varphi N_{k,t}^{1/\eta}\frac{\partial N_{k,t}}{\partial W_{k,t}} = \epsilon_w \varphi N_{k,t}^{1/\eta}\frac{N_{k,t}}{W_{k,t}}. \tag{65}$$

In symmetric equilibrium,

$$W_{k,t} = W_t, \qquad N_{k,t} = N_t, \qquad \Pi_{k,t}^w = \Pi_t^w.$$

Combining the wage-income term, the labor-disutility term, and the current and future adjustment-cost terms, and then taking a first-order approximation around a zero-wage-inflation steady state, gives

$$\pi_t^w - \beta \mathbb{E}_t \pi_{t+1}^w = \kappa_w \left[\varphi N_t^{1/\eta} - \frac{\epsilon_w - 1}{\epsilon_w}\frac{W_t}{P_t}C_t^{-\sigma}\right], \tag{66}$$

where

$$\pi_t^w \simeq \log W_t - \log W_{t-1}$$

is wage inflation and $\kappa_w > 0$ is the slope of the wage Phillips curve implied by the wage-adjustment cost, the elasticity of labor demand, and the steady-state normalization.

Define the steady-state gross wage markup as

$$\mu_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}. \tag{67}$$

Therefore,

$$\frac{\epsilon_w - 1}{\epsilon_w} = \frac{1}{\mu_w}. \tag{68}$$

Rearranging Equation (66) gives the wage New-Keynesian Phillips curve:

$$\pi_t^w = \kappa_w \left[\varphi N_t^{1/\eta} - \frac{1}{\mu_w}\frac{W_t}{P_t}C_t^{-\sigma}\right] + \beta \mathbb{E}_t \pi_{t+1}^w. \tag{69}$$

The expression inside brackets is the wage-setting wedge. The first term is the marginal disutility of supplying aggregate labor, while the second term is the marginal-utility value of the real wage, adjusted for the desired wage markup. In the absence of nominal wage rigidity, this wedge would be zero.

With flexible prices, the real wage equals marginal product:

$$\frac{W_t}{P_t} = X_t. \tag{70}$$

Consequently, the residual imposed in the numerical model is

$$0 = \kappa_w \left[\varphi N_t^{1/\eta} - \frac{1}{\mu_w} X_t C_t^{-\sigma} \right] + \beta \mathbb{E}_t \pi_{t+1}^w - \pi_t^w. \tag{71}$$

In the perfect-foresight transition exercises, $\mathbb{E}_t \pi_{t+1}^w$ is implemented as the subsequent realized wage-inflation rate.

Finally, because $W_t / P_t = X_t$, gross wage inflation satisfies

$$\frac{W_t}{W_{t-1}} = \frac{P_t}{P_{t-1}} \frac{X_t}{X_{t-1}}. \tag{72}$$

In first-order log-linear terms,

$$\pi_t^w = \pi_t + \Delta \log X_t. \tag{73}$$

Productivity is held fixed in the fiscal experiments, implying

$$\pi_t^w = \pi_t. \tag{74}$$

At a zero-inflation steady state, Equation (69) reduces to

$$\varphi \left(N^{ss}\right)^{1/\eta} = \frac{1}{\mu_w} X^{ss} \left(C^{ss}\right)^{-\sigma}. \tag{75}$$

The benchmark value $\varphi = 1.3962$ is selected to satisfy this condition. Under this value, the steady-state wage-setting residual is below 10^{-10} throughout the public-debt grid.

D Quantitative Model

This appendix provides more details regarding the results in Section 3 of the paper.

D.1 Transition Timing

D.1.1 Impact period

At impact, the government spending shock raises public demand directly. Under the benchmark deficit-on-impact rule, taxes do not move contemporaneously:

$$\Delta T_0 = 0.$$

Instead, the government absorbs the shock through higher debt issuance:

$$\Delta B_0 = \Delta G_0.$$

Thus the impact response is characterized by:

1. an exogenous increase in public demand,
2. higher output and labor through the production relation $Y_t = X_t N_t$,
3. higher household labor income,
4. no contemporaneous tax increase,
5. higher public debt on impact.

D.1.2 Subsequent periods

From period $t \geq 1$ onward, taxes rise because the fiscal rule stabilizes the higher debt stock. The linearized debt dynamics can be written approximately as

$$\Delta B_t \approx (1 + r^{ss})\Delta B_{t-1} + B^{ss}\Delta r_t + \Delta G_t - \Delta T_t.$$

This expression highlights two reasons why subsequent tax adjustment can be stronger in the high-debt state:

1. the lagged debt deviation ΔB_{t-1} is larger when debt accumulates more on impact;
2. the same interest-rate movement is more fiscally relevant when the pre-existing stock B^{ss} is larger.

Hence, even though the benchmark rule is deficit-financed on impact, it is not tax-free. Taxes are delayed, not removed.

D.1.3 Income support versus fiscal drag

A useful reduced-form representation of the consumption response is

$$\Delta C_t = \underbrace{\alpha_t(B)\,\Delta Y_t}_{\text{income support}} - \underbrace{\psi_t(B; \Delta T_{0:\infty}, \Delta r_{0:\infty}, \Delta B_{0:\infty})}_{\text{fiscal and intertemporal drag}}.$$

The term $\alpha_t(B)$ captures the extent to which households convert the income generated by the boom into current spending. The term $\psi_t(\cdot)$ captures the negative effect of taxes, future taxes, debt accumulation, and the interest-rate path.

The high-debt state differs from the low-debt state primarily because $\alpha_t(B)$ is smaller: households have lower MPCs and iMPCs and therefore provide less positive consumption support to the same fiscal shock. The negative financing component remains present, so net consumption is less supportive in the high-debt state.

D.2 Impulse responses under alternative fiscal rules

This appendix reports the state-dependent impulse responses under the alternative fiscal rules used in the paper.

D.2.1 Tax-financed rule

Figure 22 shows the responses under the tax-financed rule. The shock is financed immediately with taxes, so debt does not move on impact. Relative to the benchmark deficit-on-impact case, the tax-financed experiment produces lower multipliers and much less state dependence.

D.2.2 Partial-deficit rule

Figure 23 reports the responses under the partial-deficit rule. The impact financing split is half taxes and half debt. The resulting state dependence lies between the tax-financed and deficit-on-impact cases.

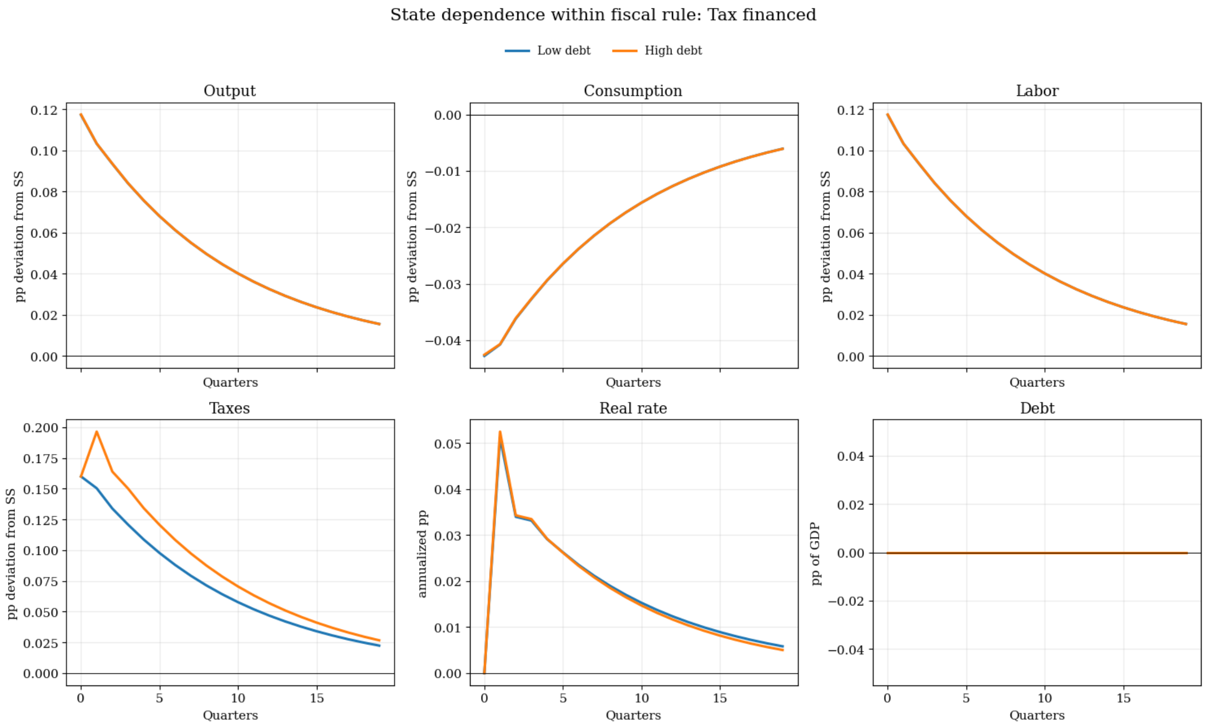


Figure 22: State dependence under the tax-financed rule. The government spending shock is financed immediately with taxes, so debt does not rise on impact. Output and labor responses are nearly identical across debt states, and the low-versus-high debt gap is much smaller than in the benchmark deficit-on-impact case.

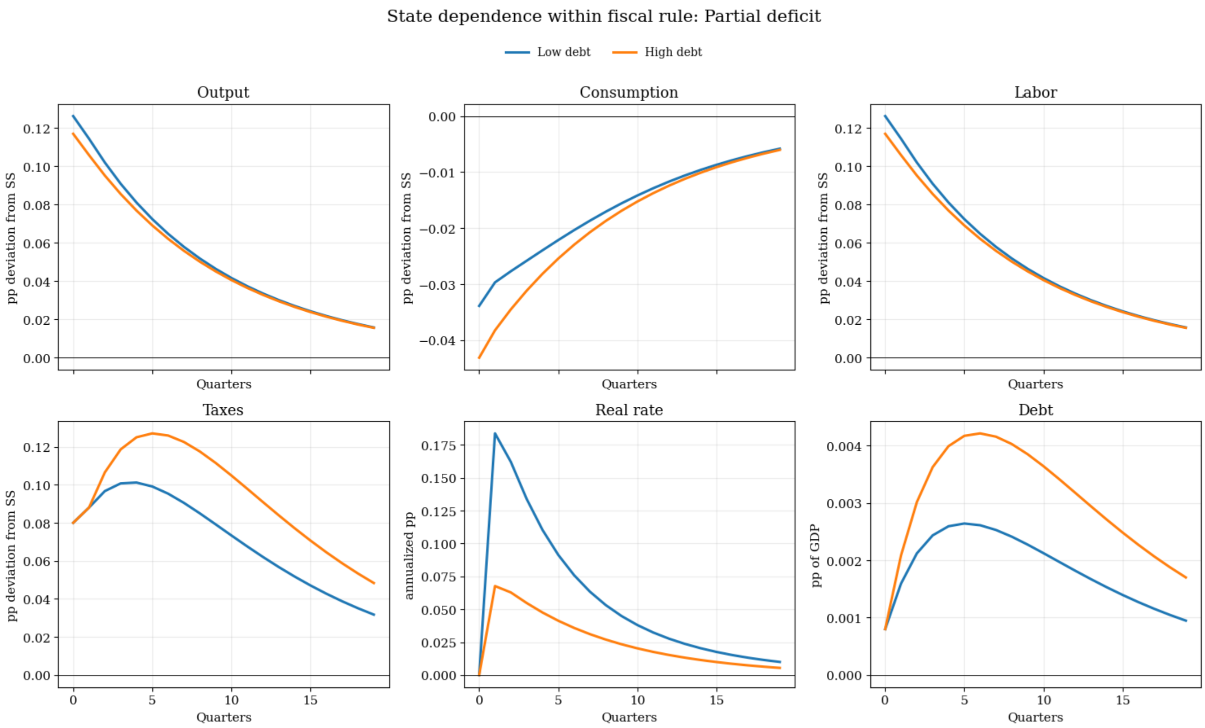


Figure 23: State dependence under the partial-deficit rule. The shock is financed partly with taxes and partly with debt on impact. The resulting gap between low-debt and high-debt responses is intermediate between the tax-financed and deficit-on-impact cases.

D.3 Additional remarks on state dependence

The benchmark model implies that the state dependence of fiscal multipliers is strongest when the spending shock is initially financed with debt and weakest when it is immediately financed with taxes. This result is consistent with the logic of the Intertemporal Keynesian Cross: the financing rule determines the sequence of net fiscal impulses $(dG_t - dT_t)$, while the debt state determines the iMPC matrix through the steady-state real rate and household balance-sheet positions. When taxes adjust immediately, there is less room for differences in the iMPC structure to matter quantitatively. When debt absorbs the shock first and taxes rise later, state dependence in private spending has more time to affect the transition.

D.4 Additional details on the wedge between MPC and multiplier state dependence

This section provides additional details on why the state dependence of fiscal multipliers is much flatter than the state dependence of the static aggregate MPC.

A useful reduced-form scalar analogy to the full IKC equation is

$$dY = dG - m\,dT + m\,dY, \tag{76}$$

which implies

$$\mu(m) \equiv \frac{dY}{dG} = \frac{1 - \lambda m}{1 - m}, \qquad \lambda \equiv \frac{dT}{dG}. \tag{77}$$

In this expression, m plays the role of a scalar spending-propensity proxy for the richer iMPC matrix M. The formula illustrates the key offset at the heart of the results. When m falls, the amplification term $1/(1 - m)$ becomes smaller, which lowers the multiplier. But the tax-drag term λm also becomes smaller, which works in the opposite direction. Therefore, even large changes in household spending propensities need not generate equally large changes in fiscal multipliers.

In the benchmark quantitative results, this is exactly what happens. The low-versus-high debt gap in the static aggregate MPC is -0.39, while the corresponding impact-multiplier gap is only -0.12. The gap in the four-quarter iMPC object is -0.31, while the four-quarter cumulative multiplier gap is only -0.11. Thus, the multiplier inherits the sign of the MPC state dependence but with a much flatter slope.

The benchmark tax timing block explains why later taxes remain central even under deficit-on-impact financing. Although $T_0 = 0$ in both states, taxes in quarters 1 to 3 are 0.17 in low debt and 0.20 in high debt, and peak taxes are larger in the high-debt state. Hence, the benchmark rule is properly interpreted as "debt first, taxes later," not as "no taxes."

Finally, the benchmark chronology confirms that the shock sequence is the same across states up to propagation. The government spending path is identical by construction, output and labor rise immediately, and the difference across states appears through the weaker private-spending feedback in the high-debt economy. That weaker feedback is the quantitative manifestation of lower MPCs and iMPCs in the high-debt state.

E Proofs and Implementation Details for Section 6

E.1 Regularity conditions

The main text states the economic assumptions. This appendix collects the regularity conditions needed for the derivative and integration-by-parts arguments.

Assumption 1 (Response and equilibrium regularity)**.** On every smooth subinterval of the debt locus: (i) $B \mapsto \Theta(B)$ is continuously differentiable; (ii) $m_h(x;\Theta)$ is bounded and continuously differentiable in Θ, with an integrable envelope for $\nabla_\Theta m_h$; and (iii) $a \mapsto m_h(e,a;\Theta)$ is of bounded variation for each e.

Assumption 2 (Distributional regularity)**.** For each earnings state e, $B \mapsto F_B(\cdot \mid e)$ is weakly differentiable on smooth subintervals, the earnings-state marginal $\pi(e)$ is fixed, and the boundary terms required by Lebesgue–Stieltjes integration by parts are either zero under the maintained regularity conditions or explicitly accounted for in the discrete summation-by-parts implementation. Wherever the monotone-absorption diagnostic is invoked, $\partial_B F_B(a \mid e) \leq 0$.

Assumption 3 (Asset-demand slope)**.** The household asset-demand map $A(r, Z)$ is continuously differentiable, $A_r > 0$, $1 + rA_Z > 0$, and $A_r - BA_Z > 0$ on the smooth debt range under consideration.

Assumption 4 (IKC operator)**.** Whenever the multiplier sign result is invoked, $\mathbf{M}(B)$ is a nonnegative bounded operator with $\|\mathbf{M}(B)\| < 1$, the fiscal path is fixed with respect to B, and $q = d\mathbf{G} - d\mathbf{T} \geq 0$.

E.2 Proofs of the main results

Proof of Proposition 1. Rearranging (32) gives

$$\big(\mathbf{I} - \mathbf{M}(B)\big)d\mathbf{Y} = d\mathbf{G} - \mathbf{M}(B)d\mathbf{T}.$$

The norm condition implies that $\mathbf{I} - \mathbf{M}(B)$ is invertible, with inverse given by the convergent Neumann series. This proves (33). To obtain (34), write

$$d\mathbf{G} - \mathbf{M}d\mathbf{T} = (\mathbf{I} - \mathbf{M})d\mathbf{T} + (d\mathbf{G} - d\mathbf{T})$$

and premultiply by $(\mathbf{I} - \mathbf{M})^{-1}$. □

Proof of Lemma 1. Define $\mathcal{H}_h(\Theta, \Psi) = \int m_h(x; \Theta) d\Psi(x)$. Along the equilibrium locus, $\mathcal{M}_h(B) = \mathcal{H}_h(\Theta(B), \Psi_B)$. The chain rule for a differentiable map into a space of finite signed measures gives

$$\frac{d\mathcal{M}_h(B)}{dB} = D_\Theta \mathcal{H}_h(\Theta(B), \Psi_B)[\Theta'(B)] + D_\Psi \mathcal{H}_h(\Theta(B), \Psi_B)[\Psi'_B].$$

Under Assumptions 1–2, the first derivative is $\int \nabla_\Theta m_h \cdot \Theta' d\Psi_B$ and the second is $\int m_h d\Psi'_B$, yielding (36). □

Proof of Lemma 2. Differentiate asset-market clearing:

$$A_r r'(B) + A_Z Z'(B) = 1.$$

The fiscal closure $T(B) = r(B)B + G$, together with fixed pretax income, implies

$$Z'(B) = -(r(B) + Br'(B)).$$

Substituting gives

$$A_r r'(B) - A_Z(r(B) + Br'(B)) = 1,$$

or

$$(A_r - BA_Z)r'(B) = 1 + r(B)A_Z.$$

Under the sign conditions in (41), division gives

$$r'(B) = \frac{1 + r(B)A_Z}{A_r - BA_Z} > 0.$$

The expression for $Z'(B)$ follows from the fiscal closure. If $A_Z = 0$, the formula reduces to $r'(B) = 1/A_r$. $\square$

Proof of Proposition 2. Equation (45) follows by substituting (42) into the definition $\mathcal{P}_{r,h} = r'\mathbb{E}_{\Psi_B}[\partial_r m_h]$. Holding the numerator and the household sensitivity fixed, the magnitude is decreasing in the positive effective slope $A_r - BA_Z$. For dominance, the reverse triangle inequality gives

$$|\mathcal{P}_h| = |\mathcal{P}_{r,h} + \mathcal{P}_{Z,h}| \geq |\mathcal{P}_{r,h}| - |\mathcal{P}_{Z,h}|.$$

Condition (46) therefore implies $|\mathcal{P}_h| > |\mathcal{D}_h|$. $\square$

Proof of Proposition 3. Let $\Delta\Theta_\varepsilon = \Theta(B+\varepsilon) - \Theta(B)$ and $\Delta\Psi_\varepsilon = \Psi_{B+\varepsilon} - \Psi_B$. A second-order expansion of $\mathcal{H}_h$ around $(\Theta(B), \Psi_B)$ gives

$$\mathcal{H}_h(\Theta + \Delta\Theta_\varepsilon, \Psi + \Delta\Psi_\varepsilon) - \mathcal{H}_h(\Theta, \Psi) = D_\Theta\mathcal{H}_h[\Delta\Theta_\varepsilon] + D_\Psi\mathcal{H}_h[\Delta\Psi_\varepsilon] + O(\varepsilon^2).$$

The cross term is $O(\|\Delta\Theta_\varepsilon\|\,\|\Delta\Psi_\varepsilon\|) = O(\varepsilon^2)$. Dividing the partial changes by ε and taking the limit yields the two components in Lemma 1. Because the ordering difference is exactly the cross interaction, the difference between policy-first, distribution-first, and local Shapley allocations vanishes after division by ε. At a kink, one or both first derivatives may fail to exist and the argument does not apply. $\square$

Proof of Proposition 5. From Proposition 1, with $q = d\mathbf{G} - d\mathbf{T}$ fixed,

$$d\mathbf{Y} = d\mathbf{T} + (\mathbf{I} - \mathbf{M})^{-1}q.$$

Differentiate the resolvent identity:

$$\frac{d}{dB}(\mathbf{I}-\mathbf{M})^{-1} = (\mathbf{I}-\mathbf{M})^{-1}\mathbf{M}'(B)(\mathbf{I}-\mathbf{M})^{-1}.$$

This yields (56). If $\mathbf{M}$ is nonnegative and has norm below one, its Neumann inverse is nonnegative. Hence $q \geq 0$ implies $(\mathbf{I}-\mathbf{M})^{-1}q \geq 0$. If $\mathbf{M}'(B) \leq 0$ element by element, the right-hand side of (56) is nonpositive element by element. Summation over any horizon with a positive government-spending denominator preserves the inequality for the cumulative multiplier. □

E.3 Absorption representation for the induced distributional term

This subsection provides a formal representation of the induced distributional term under additional regularity conditions. The representation is used as a numerical diagnostic in the quantitative decomposition and is not needed for the real-rate amplification result in the main text.

Let $F_B(a \mid e)$ denote the stationary conditional CDF of assets for earnings state e, and let $\pi(e)$ denote the stationary earnings-state probabilities. Define

$$\omega_B(a \mid e) \equiv -\frac{\partial F_B(a \mid e)}{\partial B}.$$

If the earnings-state marginal is fixed, $a \mapsto m_h(e, a; \Theta)$ is of bounded variation, the conditional distributions are differentiable in B, and the relevant boundary terms, including any mass at the borrowing constraint, are accounted for, then the induced distributional component can be written as

$$\mathcal{D}_h(B) = \sum_e \pi(e) \int_{\underline{a}}^{\infty} \omega_B(a \mid e)\, d_a m_h\big(e, a; \Theta(B)\big). \tag{78}$$

If $m_h(e, \cdot; \Theta)$ is absolutely continuous and there are no boundary contributions, this becomes

$$\mathcal{D}_h(B) = \sum_e \pi(e) \int_{\underline{a}}^{\infty} \omega_B(a \mid e)\, \partial_a m_h\big(e, a; \Theta(B)\big)\, da. \tag{79}$$

In the numerical implementation, I use the discrete summation-by-parts analogue of this formula. This is important because the stationary distribution may contain mass points on the asset grid, including at the borrowing constraint. The Stieltjes/summation-by-parts calculation is therefore checked against the independently computed hybrid-distribution counterfactual.

E.4 Full finite accounting identities

This appendix provides the full accounting underlying Proposition 4. For two debt steady states,

$$LL = \mathcal{H}^{0:7}(\Theta_L, \Psi_L), \qquad HL = \mathcal{H}^{0:7}(\Theta_H, \Psi_L),$$

$$LH = \mathcal{H}^{0:7}(\Theta_L, \Psi_H), \qquad HH = \mathcal{H}^{0:7}(\Theta_H, \Psi_H).$$

The policy-first ordering is

$$HH - LL = (HL - LL) + (HH - HL), \tag{80}$$

while the distribution-first ordering is

$$HH - LL = (LH - LL) + (HH - LH). \tag{81}$$

Equivalently,

$$\begin{aligned} HH - LL = {} & (HL - LL) + (LH - LL) \\ & + \underbrace{(HH - HL - LH + LL)}_{\text{ordering interaction}}. \end{aligned} \tag{82}$$

Averaging the two orderings gives

$$S_P = \frac{1}{2}\big[(HL - LL) + (HH - LH)\big],$$

and

$$S_D = \frac{1}{2}\big[(LH - LL) + (HH - HL)\big].$$

Proof of Proposition 4. Equations (80) and (81) follow by adding and subtracting the corresponding hybrid corner. Averaging the policy-function contribution across the two orderings gives S_P, and averaging the distributional contribution gives S_D. Their sum is exactly $HH - LL$. $\square$

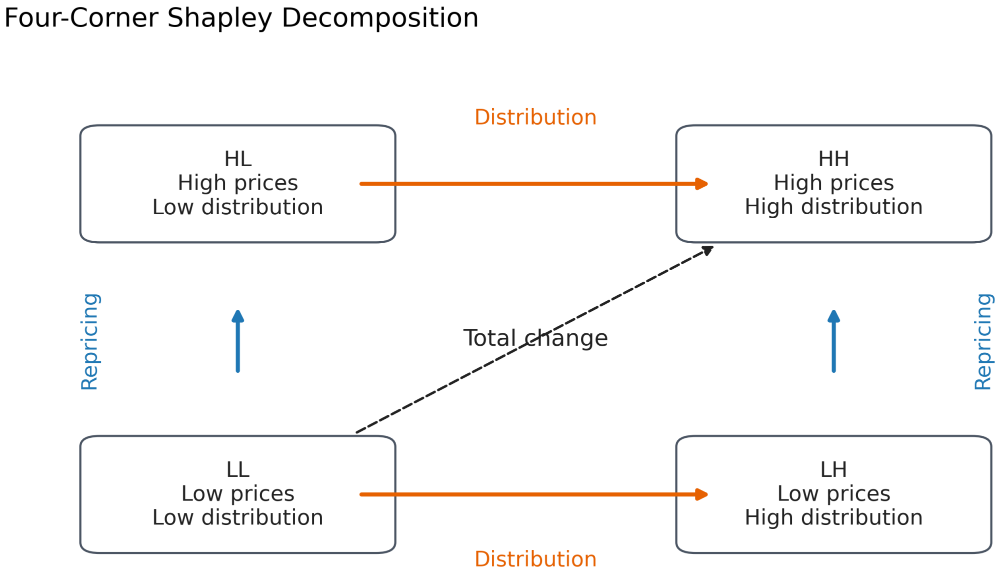


Figure 24: Four-corner decomposition. The four objects combine the low- and high-debt household environments with the low- and high-debt stationary distributions. Horizontal movements change the distribution at a fixed household environment; vertical movements change the household environment at a fixed distribution.

The finite decomposition is exact. Its allocation can nevertheless depend on ordering when the interaction term in (82) is large. The main benchmark interval has an interaction equal to only 0.6% of the total change. The local derivative decomposition is the convention-free limit on smooth subintervals.

E.5 Asset absorption and transmission incidence

High-asset positions absorb 90.0% of the additional claims and middle-asset positions absorb another 9.1%. Constrained households absorb essentially none. This accounting differs from the household incidence of the iMPC decomposition: the households absorbing the claims need not be the households whose response schedules or distributional weights generate the decline in aggregate iMPCs.

E.6 Numerical implementation

The numerical procedure mirrors the theoretical objects.

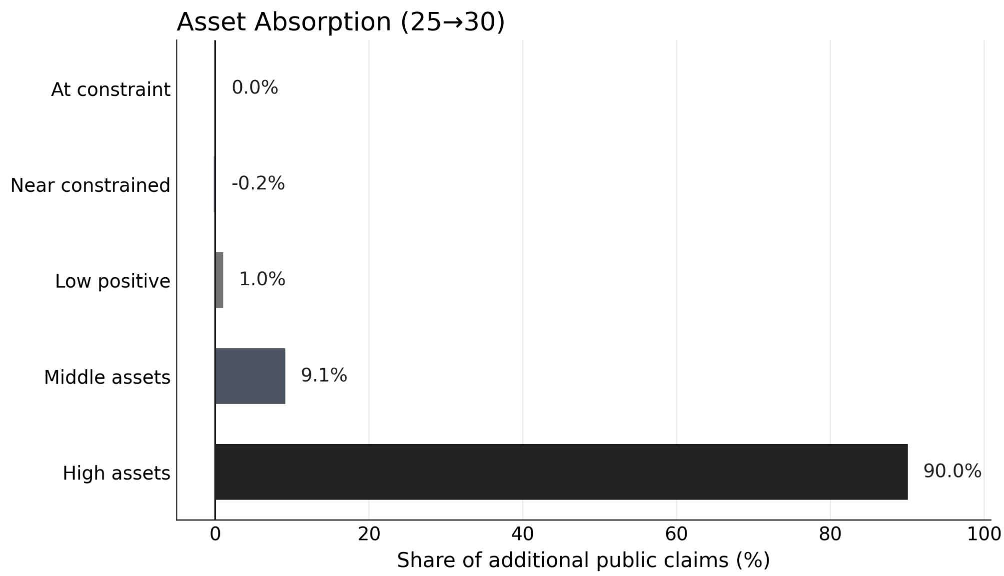


Figure 25: Absorption of additional public safe claims. Each bar reports the change in actual asset holdings of an asset-status group as a share of the increase in public debt. These are changes in asset quantities, not changes in population mass.

Equilibrium locus. For each debt target B, the full steady state is solved and market clearing imposes $A(r(B), Z(B)) = B$. Kink points associated with discrete movements across the borrowing constraint are identified separately and excluded from local derivative attribution.

True partial asset-demand derivatives. The partial derivative A_r is computed by re-solving the household stationary problem at $(r + \varepsilon_r, Z)$ and $(r - \varepsilon_r, Z)$, holding Z fixed and recomputing the stationary distribution. Similarly, A_Z is computed from $(r, Z + \varepsilon_Z)$ and $(r, Z - \varepsilon_Z)$, holding r fixed. Thus A_r and A_Z are true partial derivatives, not differences along the equilibrium debt locus, where both inputs change. The implied value of $r'(B)$ in (42) is compared with a central difference of the solved equilibrium locus.

Direct and induced components. The hybrid objects in (48)–(49) are computed by resolving household policies under swapped price inputs and by aggregating them with a fixed stationary distribution. The direct term is then split into r-only and Z-only components. The split is required to reconstruct the complete direct counterfactual to numerical precision.

Finite-difference stability. Household fake-news Jacobians are computed using two-sided finite differences with step size $h = 10^{-6}$. Stability is assessed by comparing the results with $h = 10^{-5}$; flagged or kink-adjacent points are additionally checked at $h = 5 \times 10^{-7}$. Debt intervals containing unresolved constraint-release points are excluded from headline decompositions. The selected 25%–30% comparison has stable endpoints, negligible ordering interaction, and no unresolved release contamination.

Absorption representation. For each earnings state, the absorption weight is computed from central differences of the conditional CDF. The discrete analogue of (78) uses summation by parts. The resulting distribution component is compared with the independent hybrid-distribution calculation. In the validated implementation, the two coincide to numerical precision; the weights integrate to $dA/dB = 1$ up to grid error, and the conditional CDFs exhibit no first-order-stochastic-dominance violations at the reliable smooth nodes.

Interpretation of shares. The 25%–30% Shapley allocation is reported as a representative quantitative benchmark, not as a structural constant. Finite allocations can vary with the selected debt interval and, when the interaction is material, with the ordering convention. The appendix therefore reports the decomposition across alternative intervals and cumulative horizons. The robust conclusion is that the two channels are quantitatively comparable for the cumulative iMPC, with a modestly larger distributional contribution over the validated benchmark intervals.

E.7 Additional conditional implications

The next corollary formalizes the high-debt intuition under explicit conditions. It is not needed for the main empirical or quantitative claims.

Corollary 1 (A conditional high-debt limit)**.** *Consider a sequence of smooth debt levels B_n such that: (i) the numerator $1 + r(B_n)A_Z(B_n)$ is bounded; (ii) $A_r(B_n) - B_n A_Z(B_n) \to \infty$; (iii) $\left|\mathbb{E}_{\Psi_{B_n}}[\partial_r m_h]\right|$ is bounded; (iv) $\mathcal{P}_{Z,h}(B_n) \to 0$; and (v) $|\mathcal{D}_h(B_n)|$ is bounded away from zero. Then*

$$\frac{|\mathcal{P}_h(B_n)|}{|\mathcal{D}_h(B_n)|} \longrightarrow 0. \tag{83}$$

Proof. Conditions (i)–(iii) and (42) imply $\mathcal{P}_{r,h}(B_n) \to 0$. Condition (iv) implies $\mathcal{P}_h(B_n) = \mathcal{P}_{r,h}(B_n) + \mathcal{P}_{Z,h}(B_n) \to 0$. Dividing by a denominator bounded away from zero gives (83). □

This corollary makes clear why high-debt distribution dominance is not automatic. It requires that the effective asset-demand slope diverge, that the direct income channel vanish, and that the induced distributional term remain quantitatively relevant.

F Robustness and Further Analysis

F.1 Taxes or real-rate repricing?

The household environment changes through both the equilibrium real return r and the after-tax-income shifter Z. I separate these forces using an exact two-factor Shapley decomposition of the policy-function contribution:

$$S_P = S_{P,r} + S_{P,Z}.$$

The first component changes r while holding Z fixed; the second changes Z while holding r fixed.

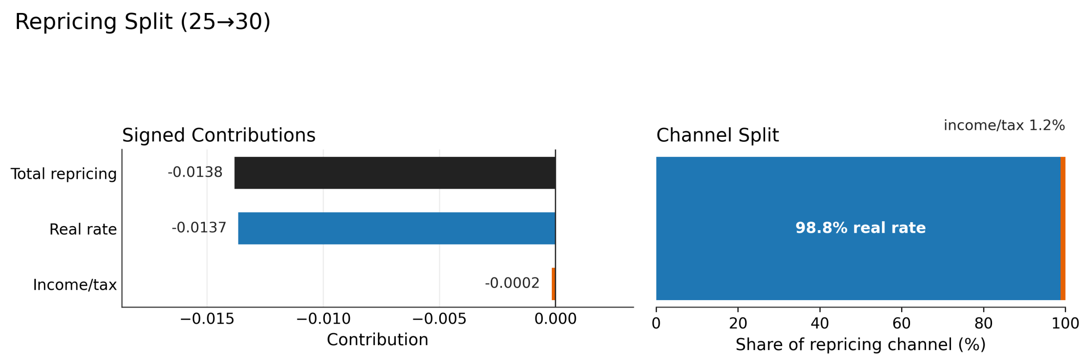


Figure 26: Real-rate and income components of repricing. Exact decomposition of the policy-function channel for the benchmark finite comparison.

The real-rate component accounts for 98.8% of the benchmark policy-function contribution, while the income/tax component accounts for 1.2%. Across the validated comparison intervals, the real-rate share ranges from 98.0% to 99.9%. This does not imply that the real rate accounts for 98.8% of the total cumulative-iMPC decline: the percentage applies only to the policy-function channel, while the distributional channel remains quantitatively important.

F.2 Horizon robustness

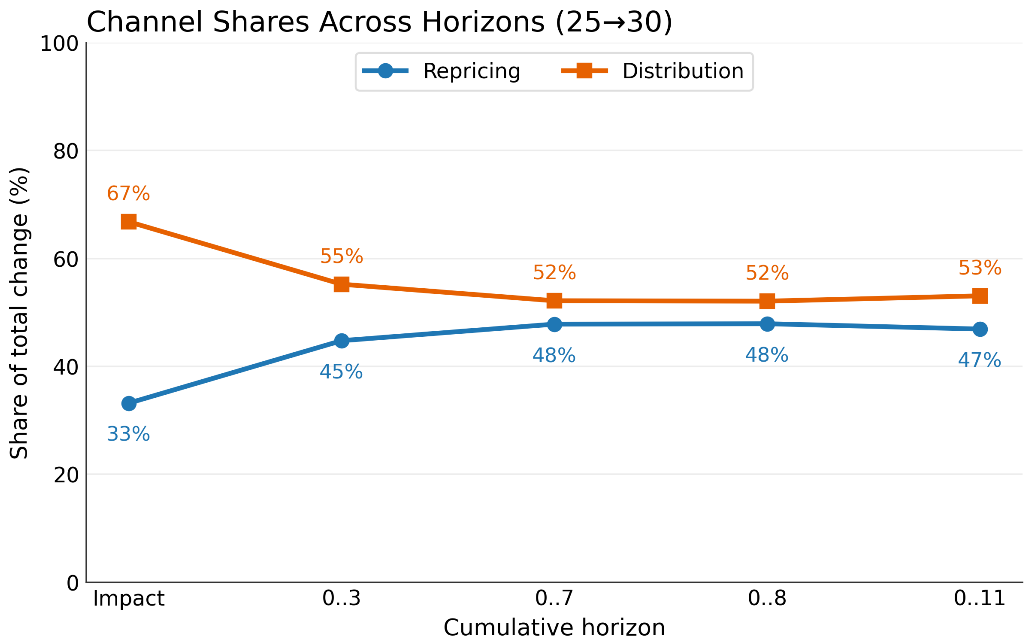


Figure 27: Channel shares across cumulative horizons. Exact Shapley shares for the benchmark comparison.

The distributional channel is larger at impact. As the cumulative horizon expands, the policy-function contribution becomes more important and the two shares converge toward an approximately even split. For the cumulative object over $t = 0, \ldots, 7$, the shares are 48% and 52%. The main text therefore always states the horizon attached to the quantitative decomposition.

F.3 Robustness across debt intervals

Across the comparisons, the policy-function share ranges from 45% to 49%, while the distributional share ranges from 51% to 55%. Both theoretically identified channels contribute materially, with a slightly larger distributional contribution in the reported comparisons.

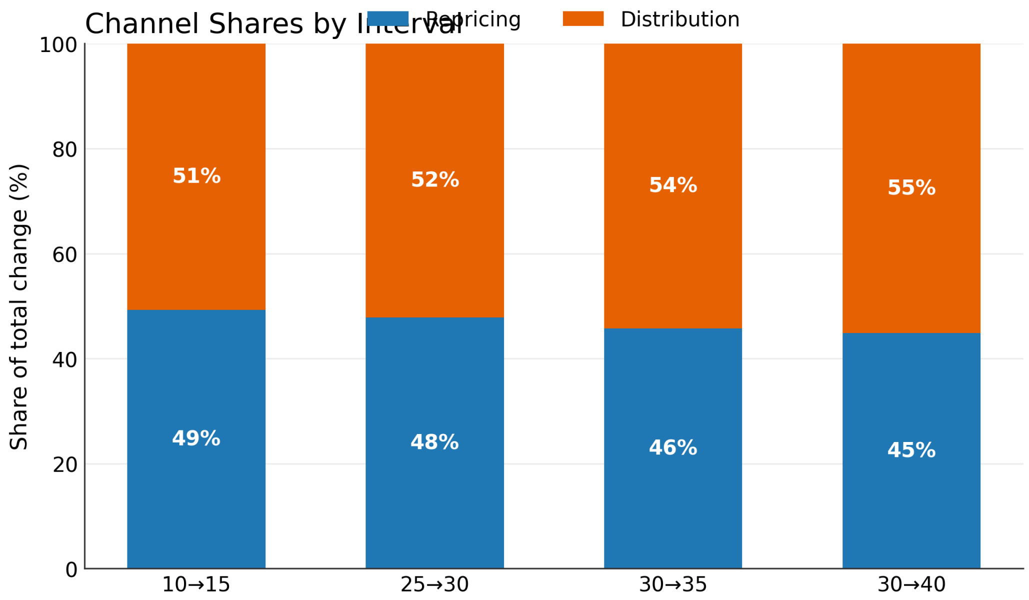


Figure 28: Channel shares across representative debt intervals. Exact Shapley decomposition of the eight-quarter cumulative iMPC over $t = 0, \ldots, 7$.

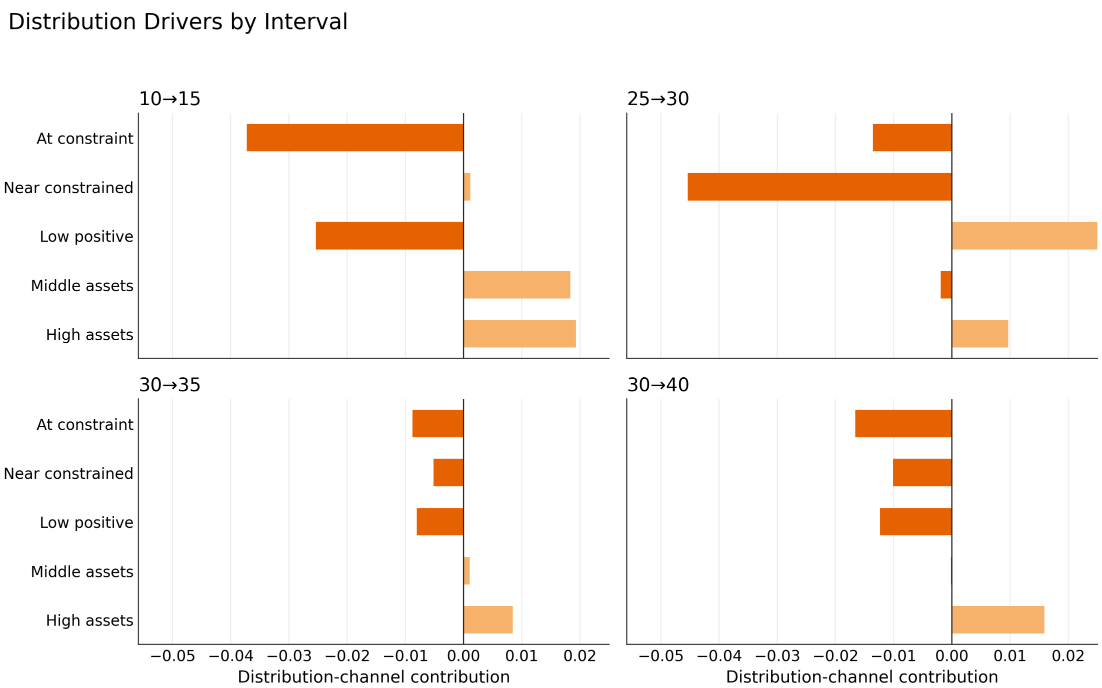


Figure 29: Household contributions to the distributional channel. Negative contributions lower the cumulative iMPC; positive contributions offset the decline.

F.4 Household incidence across intervals

The largest individual subgroup varies with the debt interval. In some comparisons, households exactly at the borrowing constraint contribute most; in others, the largest contribution comes from the near-constrained band. The robust conclusion is therefore that the distributional effect is driven by the constrained region.

G Additional Quantitative Results

This appendix documents a set of robustness exercises that complement the quantitative analysis in the main text. All experiments are conducted in the same model with sticky wages presented in Section 3, keeping the calibration of preferences, wage rigidity and monetary policy fixed. The focus is on how the transmission of fiscal policy depends on the level of public debt, on the fiscal financing rule, on the nature of the shock, and on the persistence of the spending disturbance.

Throughout, I denote by B the real stock of government debt in units of quarterly output Y, so that the conventional debt–to–annual–GDP ratio is

$$\frac{B}{Y_{\text{annual}}} = \frac{B}{4Y}.$$

For each steady state with a different B, I recompute the stationary equilibrium of the full HANK model and then solve for the transition dynamics following small unanticipated fiscal shocks.

G.1 Baseline deficit-finance rule

In the baseline specification used in Section 5, government purchases G_t follow an AR(1) process around their steady-state value G^{ss} and are financed through a combination of debt issuance and a gradual adjustment of lump-sum taxes. Linearizing the government budget constraint around the steady state implies

$$\Delta T_t \approx r^{ss}\Delta B_{t-1} + \phi_\tau\big(B_{t-1} - B^{ss}\big), \tag{84}$$

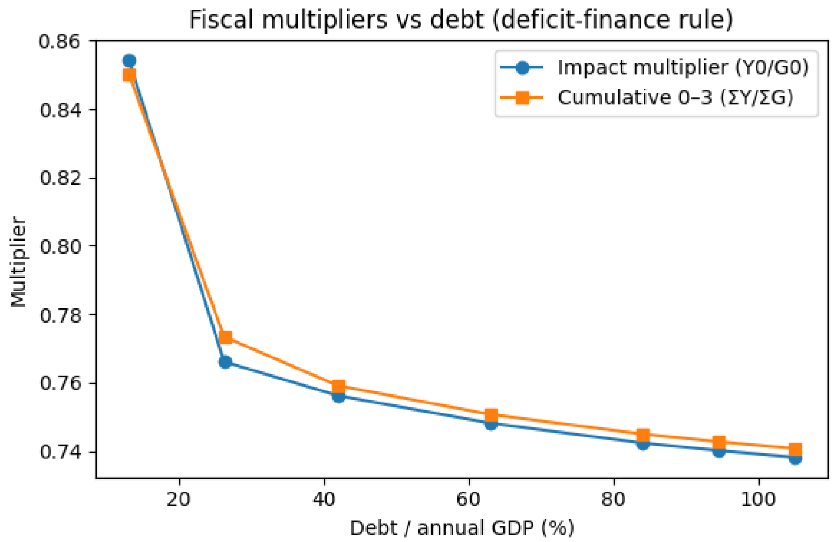


Figure 30: Impact and short-run cumulative spending multipliers as a function of the debt-to-annual-GDP ratio under the baseline deficit-finance rule.

where T_t denotes lump–sum taxes, r^{ss} is the steady-state real interest rate, and $\phi_\tau > 0$ governs the speed of fiscal consolidation.

For each B on a grid between roughly 12.5% and 105% of annual GDP, I recompute the steady state and then simulate the impulse response to a small positive shock to G_t with persistence $\rho_G = 0.9$. For a given experiment, I define the impact and short-run cumulative spending multipliers as

$$m_G^{\text{impact}}(B) = \frac{\Delta Y_0}{\Delta G_0}, \qquad m_G^{0\text{–}3}(B) = \frac{\sum_{t=0}^{3} \Delta Y_t}{\sum_{t=0}^{3} \Delta G_t}. \tag{85}$$

Figure 30 shows that both the impact and the 0–3 cumulative multipliers decline monotonically with B/Y_{annual}. Moving from low to high public debt substantially reduces the potency of government spending shocks. This behaviour is precisely the state dependence emphasized in the main text.

To connect these multipliers to household behaviour, it is useful to contrast the simple cross-sectional marginal propensity to consume (MPC) in the steady state with "effective" MPCs inferred from the fiscal experiment. Let $\text{MPC}^{ss}(B)$ denote the average impact MPC out of a small, one-period transfer in the stationary equilibrium with debt B. For the spending experiment I define

$$\text{MPC}^{G,\text{impact}}(B) = \frac{\Delta C_0}{\Delta G_0}, \qquad \text{MPC}^{G,0\text{–}3}(B) = \frac{\sum_{t=0}^{3} \Delta C_t}{\sum_{t=0}^{3} \Delta G_t}. \tag{86}$$

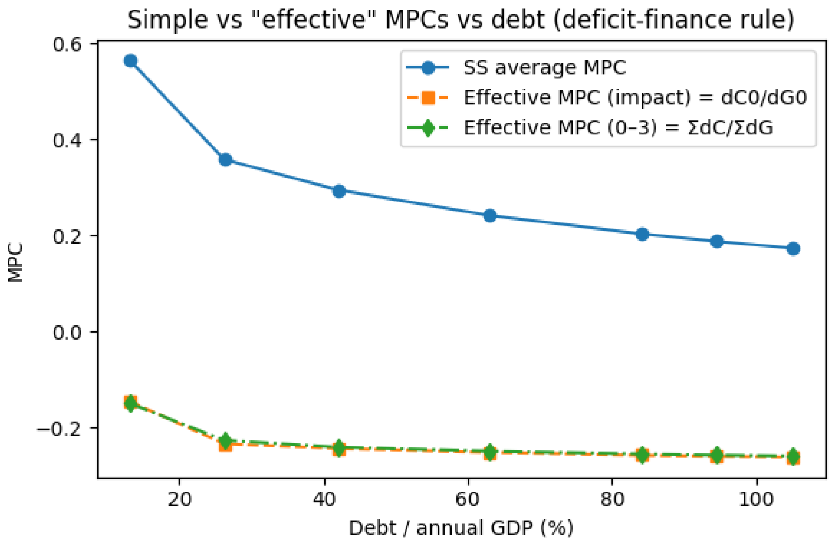


Figure 31: Simple steady-state MPC and effective MPCs (impact and 0–3 cumulative) associated with a G-shock, as a function of the debt-to-annual-GDP ratio.

Figure 31 documents two facts. First, the simple steady-state MPC $\text{MPC}^{ss}(B)$ declines sharply with the level of debt, reflecting the wealth-insurance mechanism described in the main text: richer households are less constrained and have lower iMPCs. Second, the effective MPCs associated with the G-shock are negative and exhibit only a modest dependence on B. The G-shock in this environment eventually raises lump–sum taxes and depresses the present value of disposable income, so that the net path of consumption is slightly below the baseline even though output increases in the short run. In other words, the G-shock is not a pure transfer to households; it combines demand and wealth effects.

The main implication for the paper is that the strong decline in $\text{MPC}^{ss}(B)$ is only partially reflected in the sensitivity of $m_G(B)$ with respect to debt. The level of public debt changes the wealth distribution and the cross-sectional MPCs sharply, but general-equilibrium forces muted the effect on the output multiplier.

G.2 Balanced-budget rule and alternative consolidation profiles

To gauge the role of the fiscal rule itself, I consider three alternative ways of financing the same spending shock.

First, I impose a strict balanced-budget rule under which the government adjusts taxes period by period to keep the stock of debt constant,

$$T_t = (1 + r_t)B_{t-1} + G_t - B_t, \qquad B_t = B^{ss}. \tag{87}$$

In this case the markup of taxes over steady state is more front-loaded, and the resulting multipliers are uniformly lower than under the baseline deficit rule. The decline of $m_G(B)$ with the level of debt remains present but is much weaker than under the deficit-on-impact benchmark, consistent with the fixed-price IKC logic that immediate tax finance leaves less room for household iMPC differences to affect output.

Second, I study a "long" debt-finance rule in which the government keeps taxes constant for ten years and only starts consolidating thereafter. Formally, I specify

$$T_t = \begin{cases} T^{ss} & \text{for } t < T_{\text{DF}}, \\ r^{ss}B^{ss} + G^{ss} + \phi_\tau\big(B_{t-1} - B^{ss}\big) & \text{for } t \geq T_{\text{DF}}, \end{cases} \tag{88}$$

with $T_{\text{DF}} = 40$ quarters. Relative to the baseline, this rule shifts the fiscal adjustment far into the future and therefore raises the impact and short-run multipliers for any given B. However, the shape of $m_G(B)$ as a function of debt is almost unchanged.

Third, I consider a "short" debt-finance rule in which the government allows debt to absorb the entire contemporaneous deficit but fully adjusts taxes one year later. The tax rule is

$$T_t = r^{ss}B^{ss} + G^{ss} + \phi_\tau\big(B_{t-4} - B^{ss}\big), \tag{89}$$

so that the consolidation occurs with a four-quarter lag. This rule produces multipliers that are intermediate between the baseline and the long debt-finance case. Once again, the dependence of $m_G(B)$ on the level of debt remains remarkably stable.

Taken together, these experiments suggest that the central state-dependent result of the paper, that higher public debt is associated with lower government-spending multipliers through the wealth/MPC channel, is robust to a range of reasonable assumptions about how quickly fiscal policy consolidates after a stimulus.

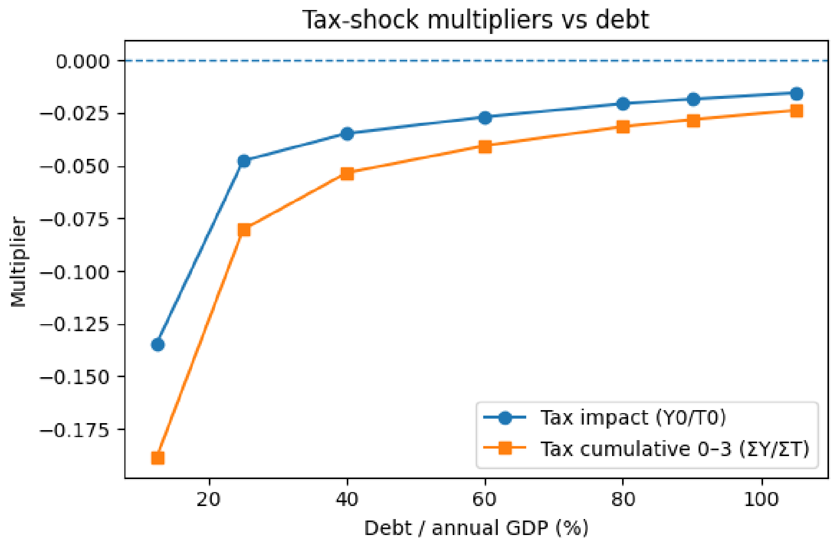


Figure 32: Impact and short-run cumulative tax multipliers as a function of the debt-to-annual-GDP ratio.

G.3 Tax shocks and fiscal consolidation

The main text focuses on spending expansions. To mimic the consolidation experiments of Brinca et al. (2021) and the tax-shock analysis in Auclert et al. (2024), I also consider shocks to taxes.

I define a small unanticipated increase in lump–sum taxes ΔT_t with persistence ρ_T, leaving G_t unchanged. For a given steady state with debt B, the impact and cumulative *tax multipliers* are

$$m_T^{\text{impact}}(B) = \frac{\Delta Y_0}{\Delta T_0}, \qquad m_T^{0\text{–}3}(B) = \frac{\sum_{t=0}^{3} \Delta Y_t}{\sum_{t=0}^{3} \Delta T_t}. \tag{90}$$

By convention, these multipliers are negative when tax increases are contractionary.

Figure 32 shows that tax hikes are strongly contractionary at low levels of debt, with relatively large negative multipliers in absolute value. As B rises, the magnitude of the tax multipliers declines. This pattern is consistent with the fact that average MPCs are higher in low-debt economies: given a unit increase in taxes, consumption of constrained agents must fall more when MPCs are large. The state dependence of tax multipliers thus mirrors, but with opposite sign, the one reported for spending shocks.

The effective MPCs associated with tax hikes, defined in analogy with (86) but using ΔT_t instead of ΔG_t, are positive and display moderate variation with B. This corroborates the idea that, for tax shocks, the simple cross-sectional MPC is a better sufficient statistic for the output effect, in line with the sufficient-statistic logic in Auclert et al. (2024).

G.3.1 Speed of fiscal consolidation

In the benchmark deficit-financed experiment in section 5.2 the government chooses taxes according to the feedback rule

$$T_t \;=\; r^* B^* + G^* + \phi_\tau \big(B_{t-1} - B^*\big), \tag{91}$$

combined with the one–period budget constraint

$$B_t \;=\; (1 + r_t) B_{t-1} + G_t - T_t. \tag{92}$$

Linearising (91)–(92) around the steady state yields the approximate law of motion

$$B_t - B^* \;\approx\; \big(1 + r^* - \phi_\tau\big)\,(B_{t-1} - B^*), \tag{93}$$

so that the effective persistence of government debt is given by

$$\rho_B \;\simeq\; 1 + r^* - \phi_\tau.$$

A larger feedback coefficient ϕ_τ therefore corresponds to *less* persistent debt and faster fiscal consolidation, whereas a smaller ϕ_τ implies a highly persistent debt process that is close to a unit root.

To assess how the speed of consolidation interacts with the level of debt, I recompute steady states and impulse responses for the deficit-finance experiment under five alternative values of the feedback parameter,

$$\phi_\tau \in \{0.05, 0.20, 0.50, 0.70, 0.90\},$$

keeping all other parameters fixed and maintaining the benchmark persistence of the government-spending shock, $\rho_G = 0.9$. For each value of ϕ_τ and each initial steady-state debt level, I solve an impulse response to a 1% AR(1) spending shock and compute the impact and four–quarter cumulative multipliers

$$M_0 = \frac{\Delta Y_0}{\Delta G_0}, \qquad M_4 = \frac{\sum_{t=0}^{3} \Delta Y_t}{\sum_{t=0}^{3} \Delta G_t}.$$

Figure 33 shows that, for any given initial debt-to-GDP ratio, a very passive rule with $\phi_\tau = 0.05$ (implied persistence $\rho_B \approx 0.96$) generates the lowest multipliers. Increasing ϕ_τ from 0.05 to 0.20 or 0.50 shifts both the impact and cumulative multipliers upward, while further increases to $\phi_\tau = 0.70$ or 0.90 mainly generate small level changes. In all cases, however, the slope of the multiplier with respect to the initial debt ratio remains negative and of similar magnitude.

The intuition is that a sluggish rule which lets additional debt linger for very long periods raises the effective real interest rate faced by households and shifts the wealth distribution towards richer, low-MPC agents, thereby dampening the demand response to the spending shock. More active rules stabilise debt more rapidly and prevent this additional crowding-out channel from operating, so the multiplier is slightly larger. At the same time, the cross-sectional response to a given spending shock is still governed mainly by the steady-state distribution of wealth; for each value of ϕ_τ the average MPC declines sharply with the level of debt, whereas the fiscal multiplier declines much more slowly.

This robustness exercise confirms that the negative relationship between household-absorbed public debt and spending multipliers does not hinge on a particular choice for the speed of fiscal consolidation. The mechanism should be interpreted through the two margins emphasized in Section 6: higher debt changes the iMPC structure through both induced balance-sheet reallocation and direct repricing, with the relative contribution depending on horizon and location along the debt locus.

G.4 Shock persistence and interaction with debt

Finally, I experiment with different degrees of persistence of government spending shocks while keeping the fiscal rule fixed. Let

$$\Delta G_t = \Delta G_0 \rho_G^t$$

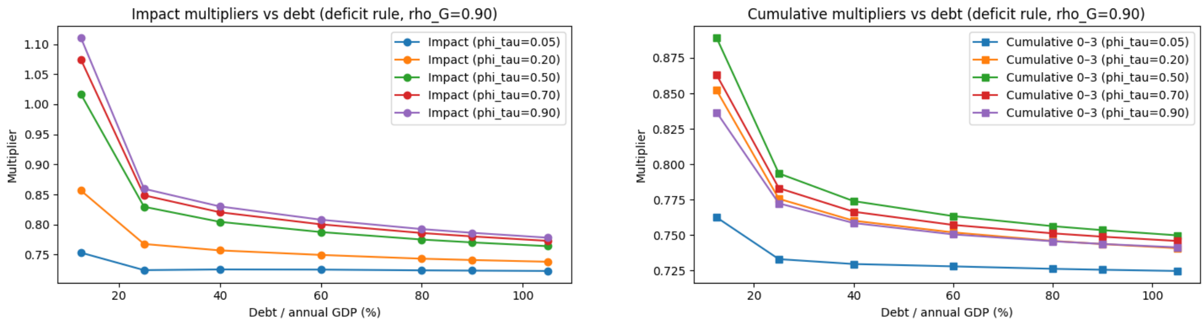


Figure 33: Impact and four–quarter cumulative multipliers as a function of the initial public-debt-to-GDP ratio for different values of the fiscal-feedback coefficient ϕ_τ.

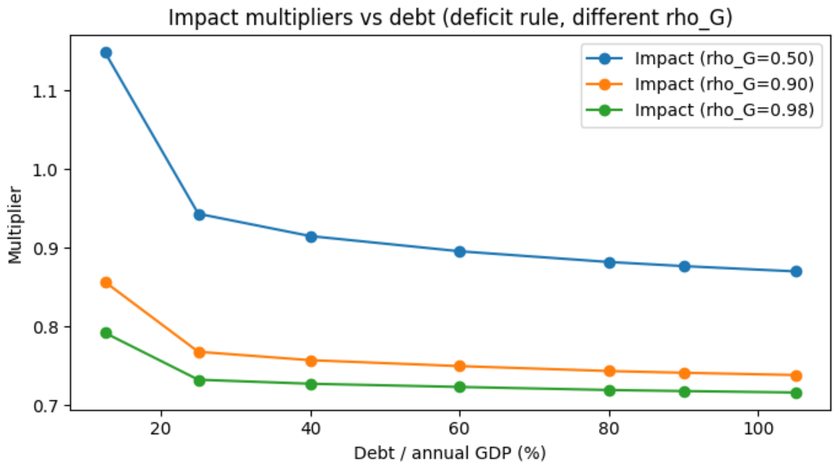


Figure 34: Impact spending multipliers for different levels of debt and different persistence parameters ρ_G of the spending shock.

with $\rho_G \in \{0.5,\ 0.9,\ 0.98\}$. For each ρ_G I recompute the impact and cumulative multipliers along the grid of debt levels.

Figure 34 shows that higher persistence raises the level of the multipliers for any given B, as expected: a more persistent shock has a larger present value and generates a stronger response of demand and of the marginal utility of income. However, even when ρ_G is close to unity, the slope of $m_G(B)$ remains negative. When combined with the fiscal rule (84), this result suggests that both the stock of debt and the consolidation parameters shape the state dependence of multipliers, but the qualitative pattern emphasized in the main text is robust to plausible changes in shock persistence.

G.5 Discussion

The exercises in this appendix reinforce the main mechanisms of the paper. Higher public debt changes household spending responses through both induced balance-sheet reallocation and direct repricing. Alternative fiscal rules, long or short debt-finance horizons, and different shock processes change the level of multipliers, but the negative relationship between household-absorbed public debt and fiscal transmission remains. The comparison between spending and tax shocks also clarifies when iMPCs are informative about output responses and when general-equilibrium feedbacks through interest rates, wages, and future taxes weaken the simple fixed-price IKC mapping.

H Comparative statics: EIS and Frisch elasticity

H.1 Elasticity of intertemporal substitution

I vary the intertemporal elasticity of substitution (EIS) around the benchmark calibration and recompute the public debt variation impact on fiscal multipliers. The purpose of this exercise is not to select a calibration in which one channel dominates, but to verify that the qualitative mechanism in Section 6 survives conventional preference variation.

The main result is robust: higher household absorption of public debt lowers the impact iMPC across the EIS values considered. The channel shares, however, are not structural constants. As the EIS changes, the slope of household asset demand changes, and therefore the real-rate movement required to absorb an additional unit of public debt also changes. This is the comparative-static logic formalized in Proposition 2.

The final decomposition robustness leads to three conclusions. First, low asset-demand elasticity amplifies the direct real-rate channel at low debt. Second, this amplification does not imply unconditional direct-channel dominance: at impact, induced reallocation remains larger at reliable smooth nodes. Third, the composition of the direct channel is highly stable. Across the EIS values and smooth debt nodes examined, the after-tax-income component is quantitatively small, so the direct policy-function margin is primarily a real-rate margin.

I therefore interpret the EIS exercise as a robustness check on the mechanism, not as evidence for a fixed factor-price share. The robust object is the sign of the debt effect on iMPCs and the real-rate composition of the direct margin; the direct-versus-induced split varies with the horizon, the debt interval, and the local asset-demand slope.

H.2 Frisch elasticity of labor supply

I also vary the Frisch elasticity parameter η in the household block. In the current implementation, the intensive labor-supply decision does not affect the construction of the intertemporal MPCs for transfers to Z, and the household Jacobian with respect to Z is effectively invariant to φ for the steady states I study. Consistent with this, I find that for $\varphi \in \{0.2, 0.4, 0.6, 0.8\}$, the impact iMPCs at low and high debt are numerically unchanged, with $\text{iMPC}_0^{\text{impact}} \approx 0.1801$ and $\text{iMPC}_1^{\text{impact}} \approx 0.1239$.

This invariance is therefore an expected feature of the model: the Frisch elasticity affects aggregate labor and wages in general equilibrium, but does not shift the micro consumption response to a pure Z-transfer in the way that matters for the iMPC.

H.3 Intertemporal MPC matrix across steady states

This appendix quantifies how intertemporal marginal propensities to consume (iMPCs) in the model vary with the level of resident-held public debt. I follow the Intertemporal Keynesian Cross (IKC) approach of Auclert et al. (2024). For a given steady state with debt B/Y, I hold the real interest rate r and the after-tax wage–price ratio $Z \equiv W/P$ fixed at their steady-state values and perturb households' budget constraints with a small one-time increase in liquid resources at date s. Let Z_s denote this transfer. I then record the change in aggregate consumption C_t at all horizons $t \geq s$. The resulting $T \times T$ matrix

$$\Lambda_{t,s}(B) \equiv \frac{\partial C_t}{\partial Z_s} \Big/ Y^{\text{ss}}, \qquad t, s = 0, \dots, T-1,$$

is the IKC matrix of iMPCs in that steady state.[23]

I compute $\Lambda(B)$ for two steady states that differ only in the level of resident-held public debt: a low-debt economy with $B/Y \approx 10\%$ and a high-debt economy with

[23] In practice I construct $\Lambda(B)$ from the Jacobian of the household block with respect to the transfer path, evaluated in partial equilibrium at the steady-state values of r and Z for a given B/Y. I set $T = 40$, corresponding to ten years of quarterly horizons.

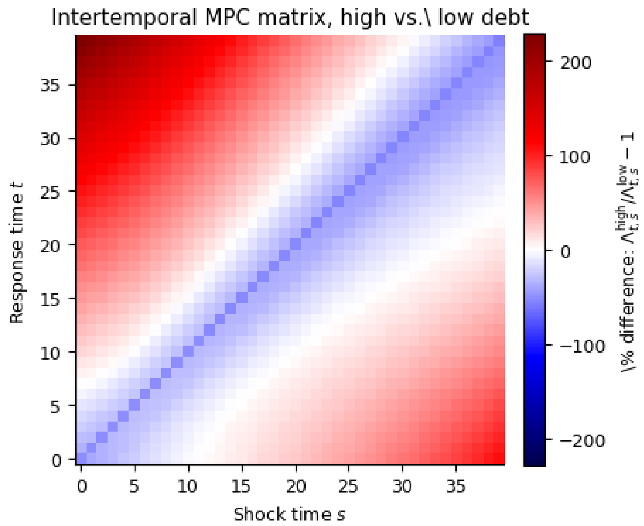


Figure 35: Percent difference in the IKC iMPC matrix between the high-debt steady state ($B/Y \approx 100\%$) and the low-debt steady state ($B/Y \approx 10\%$). The figure reports $100 \times \left(\Lambda^{\text{high}}_{t,s}/\Lambda^{\text{low}}_{t,s} - 1\right)$ for all shock dates s and response dates t. Blue regions correspond to smaller iMPCs at high debt, red regions to larger iMPCs. High debt reduces most entries of the matrix, especially those close to the diagonal (short gaps $t - s$), where the iMPCs are roughly 40–60% lower in the high-debt steady state.

$B/Y \approx 100\%$. Figure 35 plots the percent difference between the two matrices,

$$100 \times \left(\frac{\Lambda^{\text{high}}_{t,s}}{\Lambda^{\text{low}}_{t,s}} - 1 \right),$$

for all (t,s). I use a diverging color scale: blue shades indicate entries that are *smaller* in the high-debt steady state ($\Lambda^{\text{high}}_{t,s} < \Lambda^{\text{low}}_{t,s}$), while red shades indicate entries that are larger.

Two features stand out in Figure 35. First, high public debt reduces *almost all* entries of the IKC matrix: for the bulk of (t,s) I have $\Lambda^{\text{high}}_{t,s} < \Lambda^{\text{low}}_{t,s}$ (blue colors). Second, the effect is strongest for short gaps $t - s$, i.e. along and just below the diagonal. These are the horizons where households normally spend a large fraction of an income innovation, and it is precisely there that higher debt makes them much less hand-to-mouth.

To further compress the information in $\Lambda(B)$, Figure 36 reports the diagonal entries $\Lambda_{t,t}$ in the low- and high-debt steady states, together with their percent difference. The diagonal collects the response of consumption at date t to a transfer that arrives in the same period, and can be interpreted as the sequence of impact MPCs at each horizon.

Figure 36 shows that higher public debt lowers the diagonal of Λ by about one half at all horizons. In other words, conditional on a one-unit income perturbation,

Figure 36: Diagonal of the IKC matrix in the low- and high-debt steady states. Left panel: $\Lambda_{t,t}$ for $B/Y \approx 10\%$ (blue) and $B/Y \approx 100\%$ (orange). Right panel: percent change $100 \times (\Lambda^{\text{high}}_{t,t}/\Lambda^{\text{low}}_{t,t} - 1)$. For every horizon t the diagonal entry in the high-debt steady state is roughly 45–55% smaller than in the low-debt steady state, reflecting that households consume a much smaller fraction of additional income when public debt (and the real interest rate) are higher.

aggregate consumption responds less in the high-debt steady state than in the low-debt steady state. This is the steady-state counterpart of the mechanism emphasized in the main text: a higher B/Y raises the real interest rate, tilts intertemporal choices toward saving, and reduces impact iMPCs throughout the distribution.

Despite these large changes in iMPCs, the state-dependent fiscal multipliers in the model decline more modestly when B/Y increases. For example, moving from $B/Y \approx 10\%$ to $B/Y \approx 100\%$ reduces the four-quarter cumulative government-spending multiplier by about 10%, whereas the diagonal entries of Λ fall by roughly 50%. This apparent discrepancy is exactly what the IKC logic predicts. In the scalar approximation with an effective iMPC λ, the multiplier is $M(\lambda) = 1/(1-\lambda)$; halving λ from 0.2 to 0.1 lowers the multiplier from 1.25 to 1.11, that is, by about 11%. The full-matrix results mirror this nonlinearity: high public debt shrinks the entire iMPC matrix substantially, especially at short horizons, and this maps into a moderate, but economically meaningful, decline in state-dependent fiscal multipliers through the operator $(I - \Lambda)^{-1}$.